CS:4350
Logic in Computer Science

From English to Propositional Logic

Acknowledgements

- The material in this lecture is based in part on the following sources:
    - “Logic and Critical Reasoning”, Vaidya and Erikson
    - Ideas and examples from Scott Martin, Ohio State University
    - Examples from Volker Halbach, Oxford
    - Examples from Alper Ungor, University of Florida

Objectives

- Learn how to use propositional logic to model English propositional sentences.
- Understand the relationships between the English connectives with the propositional logic connectives
- Understand how to translate English statement into propositional logic form

English to Propositional Logic

Main idea for translating a collection of English sentences to a statement in propositional logic (PL)

1. Read the passage in English and determine all significant units within the passage.
   1. Identify atomic propositions
   2. Determine appropriate logical connectives based on recognizable keywords in the English sentences
2. Set a scheme of abbreviation by assigning propositional letters of PL, such as **p**, **q**, **r**,… to sentences of the passage you are translating, such as "John is tall" or "Mary is smart"
3. Using the translation guides (to be presented in this lecture), translate the sentences of English into formulas of PL

Logic Connectives

	Math Unicode	ASCII
Negation	¬	~
Conjunction	∧	^
Disjunction	∨	V
Implication	→	->
Bi-implication	↔	<->

Negation

- Negation is expressed by words or phrases like not , it is not the case that, neither … nor, etc.

Example

(1)
a. Clint went to the Chatterbox Café.
b. Clint did not go to the Chatterbox Cafe.

*If (1a) is represented as **p**,*
*Then (1b) is represented as ¬**p***

c. It is not the case that CO_2-emissions are being cut.

*Let **p** = "CO_2-emissions are being cut"*
*Then (1c) is represented as ¬**p***

Negation

- Negation is not always easy to recognize because it is often expressed in verb and adjective prefixes like un-, in-, im-, ir-, etc.

Example

1. Their action is unethical/invalid/immoral/irresponsible

All implicitly negative sentences

Their action is not ethical/valid/moral/responsible

*Let **p** = "their action is ethical/…"*
*Then (1) is represented as ¬**p***

Maximum Logical Revelation

The Principle of Maximal Logical Revelation: Always translate to reveal as much logical structure as the target language allows for

If the sentence is "John is not tall"

- set **p** to "John is tall" and translate as **¬p**
- do not set **p** to "John is not tall", since "not" can be translated out with **¬**

Source: "Logic and Critical Reasoning", Vaidya and Erikson.

Conjunction

- Conjunction often involves the word and, and these cases are typically easy to work with

Example

Chao went to Dillons and Fred went to Best Buy

*Let **p** = "Chao went to Dillons"*
*Let **q** = "Fred went to Best Buy"*
Then the claim is represented as $p \land q$

Show all the records in the data base for people that …

… are older than 25 and who live in Manhattan

*Let **p** = "Age > 25"*
*Let **q** = "City = "Manhattan""*
Then the selection criteria is represented as $p \land q$

Maximum Logical Revelation

The Principle of Maximal Logical Revelation: Always translate to reveal as much logical structure as the target language allows for

- If the sentence is "John is not tall"
  - Set **p** to "John is tall" and translate as **¬p**
  - Do not set **p** to "John is not tall", since "not" can be translated out with **¬**
- If the sentence is "John is tall and Mary is smart"
  - Set **p** to "John is tall" and **q** to "Mary is smart" and translate as **p ∧ q**
  - Do not set **p** to "John is tall and Mary is smart", since "and" can be translated out with **∧**

Source: "Logic and Critical Reasoning", Vaidya and Erikson.

Conjunction

- Both (usually together with and) can also be an indicator of conjunction

Example

Both Chao and Fred have credit cards

*Let **p** = "Chao has a credit card"*
*Let **q** = "Fred has a credit card"*
Then the claim is represented as $\boldsymbol{p \land q}$

Consider the Following

UI beat ISU in basketball, but ISU won in football.

I hiked 10 miles with a heavy backpack. Moreover, it was raining as I hiked.

Venice is a beautiful city. However, the smell of the canals is a bit distracting.

Words like *but*, *moreover*, *however* also join individual claims whose truth is asserted (i.e., they can be translated as *and*), but they also *shade* the interpretation for the listener/reader

Such shading is lost in a translation to propositional logic

Source: Inspired by “Logic and Critical Reasoning”, Vaidya and Erikson.

Consider the Following

Axel Rose"s voice went out, and the crowd threw food on the stage.

John discovered the cure for cancer and became famous.

Sometimes the use of and implies a temporal order or causality

Such aspects cannot be captured directly in propositional logic

Source: Inspired by "Logic and Critical Reasoning", Vaidya and Erikson.

Collective Subject

Jane and Bill got married.

Not quite the same thing as “Jane got married” and “Bill got married”

June, July, and August make up the summer recess.

Main point:
Sometimes (e.g., when we have a collective subject) we do not want to split things joined by and into separate propositions

Source: Inspired by “Logic and Critical Reasoning”, Vaidya and Erikson.

Conjunction

- Conjunction sometimes involves the word and, but not always.
  The following words can also be translated as conjunctions:
  - but, nonetheless, however, nevertheless, moreover, although, whereas, …

Example

(2) Pastor Ingqvist is a Lutheran but Father Wilmer is not.

Let $\boldsymbol{p}$ *= "Pastor Ingqvist is a Lutheran"*
Let $\boldsymbol{q}$ *= "Father Wilmer is a Lutheran"*
Then (2) is represented as $\boldsymbol{p \wedge \neg q}$

Adapted from Scott Martin, Ohio State University

Disjunction

- Disjunction usually involves the word or
  - but need to distinguish between *exclusive-or* and *inclusive-or*

Example (exclusive-or)

(3) a. You will either pass CS4310 or fail CS4310.

Let ***p*** *= "You will pass CS4310"*
Let ***q*** *= "You will fail CS4310"*
Then (3a) is represented as $(p \lor q) \land \neg(p \land q)$*, in other words, only one of these two propositions can be true*

Alternatively, we could infer that pass is the opposite of fail and have a single proposition **p** for pass (and **¬p** fail)

Example (inclusive-or)

(3) b. CS4310 is open in *either* Spring *or* Fall.

Let ***p*** *= "CS4310 is open in Spring semester"*
Let ***q*** *= "CS4310 is open in Fall semester"*
Then (3b) is represented as $p \lor q$*, in other words, both* ***p*** *and* ***q*** *can be true*

- In many English sentences, exclusive-or is intended
- However, in PL, the or connective is inclusive (so the exclusion condition must be explicitly added)

Consider the Following

The system shall maintain the room temperature within the target range unless a sensor fails.

The unless indicates an exceptional circumstance where the requirement to maintain the temperature does not apply

Let $\boldsymbol{p}$ *= "System maintains room temperature within target range"*
Let $\boldsymbol{q}$ *= "Sensor fails"*
Then the requirement could be represented as $\boldsymbol{p \vee q}$

Note:

- *It might be more natural to write* $\boldsymbol{\neg q \rightarrow p}$ *(sensor not failing implies system working).*
- $\boldsymbol{A \rightarrow B}$ *is equivalent to* $\boldsymbol{\neg A \vee B}$*, so* $\boldsymbol{\neg q \rightarrow p}$ *is equivalent to* $\boldsymbol{\neg\neg q \vee p}$*, which is equivalent to* $\boldsymbol{p \vee q}$

Implication

- Implication is used to capture conditionality. The following words can also be translated as implications:
  - if … then … , provided … that …, assuming, only if, given …

Example

a. Wally eats Powdermilk biscuits only if Evelyn makes them.

Let $\boldsymbol{p}$ *= "Wally eats Powdermilk biscuits"*
Let $\boldsymbol{q}$ *= "Evelyn makes them"*
Then, intuitively, $\neg \boldsymbol{q} \rightarrow \neg \boldsymbol{p}$ *which is equivalent to* $\boldsymbol{p} \rightarrow \boldsymbol{q}$

b. You will get an A if your score is above 90.

Let $\boldsymbol{p}$ *= "You will get an A"*
Let $\boldsymbol{q}$ *= "Your score is above 90"*
Then (4b) is represented as $\boldsymbol{q} \rightarrow \boldsymbol{p}$

Intuitively, this is actually an "iff" condition, so our English language characterization of the situation is not adequate. Assuming the literal reading of the sentence is the intended one, the grading policy allows A's also in situations other than scoring more than 90 (e.g., having done some extra credit work)

Adapted from Scott Martin, Ohio State University

Implication

Sentences with if can be tricky at first to translate correctly to logic.

Use the following reasoning:

- ***A*** if ***B***: If I know ***B***, I can conclude ***A*** (But if I know ***A***, I do not really know anything about ***B***) $B \rightarrow A$

- ***A*** only if ***B***: If I know ***A***, I can conclude ***B*** (Because ***B*** being true is the only way for ***A*** to be true) $A \rightarrow B$

Double implication

- Double implication makes a stronger claim than the conditional.
- The following words can be translated as double implications:
  - if and only if, just in case, exactly when, …

Example

(5) You get an A if and only if your grade is above 90

Let $\boldsymbol{p}$ *= "You get an A"*
Let $\boldsymbol{q}$ *= "Your grade is above 90"*
Then (5) is represented as $\boldsymbol{p \leftrightarrow q}$
(or, $\boldsymbol{(p \rightarrow q) \wedge (q \rightarrow p)}$*, or* $\boldsymbol{p = q}$*)*

Exercise 1

Translate each of the following sentences to propositional logic.

1. An item was not inserted into the queue.
2. An item can be removed from the queue only if the queue is non-empty.
3. A client must hold the lock on the queue to remove an item from the queue
4. The system shall ensure that the temperature is within the target range and that camera acquires an image every second.
5. The system user authentication mechanism shall provide authentication via user-id/password or via retina scan.
6. It is not the case that if the programmer position is open both Jill and Sheila will apply.
7. Only if Jill applies for the position will Jay apply.
8. Neither Sam nor Alan will apply for the position if Jill applies.

Combinations of Connectives

Example

a. Florian neither washed the car nor went to the mercantile.

Let $\boldsymbol{p}$ *= "Florian washed the car"*
Let $\boldsymbol{q}$ *= "Florian went to the mercantile"*
Then (a) is represented as $\neg(p \lor q)$

b. It"s not true that Clint owns both a Ford and a Chevy dealership.

Let $\boldsymbol{p}$ *= "Clint owns a Ford dealership"*
Let $\boldsymbol{q}$ *= "Clint owns a Chevy dealership"*
Then (b) is represented as $\neg(p \land q)$

c. Myrtle doesn't cook a walleye unless Clint catches it.

Let $\boldsymbol{p}$ *= "Myrtle cooks a walleye"*
Let $\boldsymbol{q}$ *= "Clint catches a walleye"*
Then (c) is represented as
either $\neg p \lor q$, $p \rightarrow q$, *or* $\neg q \rightarrow \neg p$

Adapted from Scott Martin, Ohio State University

Combinations of Connectives

Example

a. I met my ex-girlfriend today and either she grew taller or I got shorter.

Let $\boldsymbol{p}$ *= "I met my ex-girlfriend today"*
Let $\boldsymbol{q}$ *= "She grew taller"*
Let $\boldsymbol{r}$ *= "I got shorter"*
Then (a) is represented as $\boldsymbol{p \land (q \lor r)}$

b. You get an A only if you score at least 50% on the midterm or you submit a HW.

Let $\boldsymbol{p}$ *= "You get an A"*
Let $\boldsymbol{q}$ *= "You score at least 50% on the midterm"*
Let $\boldsymbol{r}$ *= "You submit a HW"*
Then (b) is represented as $\boldsymbol{p \rightarrow (q \lor r)}$

Combinations of Connectives

Example

d1. State the negation of “I am a doctor or a lawyer”.

d2. “I am not a doctor and I am not a lawyer.”

*Let **p** = "I am a doctor"*
*Let **q** = "I am a lawyer"*
d1: $\neg(p \lor q)$
d2: $\neg p \land \neg q$

e1. State the negation of “She is rich and beautiful.”

e2. “She is either not rich or not beautiful”.

*Let **p** = "She is rich"*
*Let **q** = "She is beautiful"*
e1: $\neg(p \land q)$
e2: $\neg p \lor \neg q$

Combinations of Connectives

Example

f. If Vettel finishes in the top 10 or Button doesn't win, then Vettel will become world champion.

Let $\boldsymbol{p}$ *= "Vettel finishes in the top 10"*
Let $\boldsymbol{q}$ *= "Button wins"*
Let $\boldsymbol{r}$ *= "Vettel will become world champion"*
Then (f) is represented as $\boldsymbol{(p \lor \neg q) \rightarrow r}$

Summary

English	Propositional Logic
A and B | A but B | A; moreover/however, B	$A \land B$
if A, then B | A implies B | A forces B	$A \rightarrow B$
A if B | A when B | A whenever B | only if A, B | B only if A	$B \rightarrow A$
A exactly/precisely when B | A if and only if B	$A \leftrightarrow B$ | $B \leftrightarrow A$
A or B (or both) | A unless B	$A \lor B$ | $\neg B \rightarrow A$
either A or B (but not both)	$A \oplus B$ | $(A \lor B) \land \neg(A \land B)$