CS:4350 Logic in Computer Science

Transition Systems

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The University of Iowa

Credits

These slides are largely based on slides originally developed by **Andrei Voronkov** at the University of Manchester. Adapted by permission.

Outline

State-changing systems

Our main interest from now on is modeling *state-changing systems*

We assume a discrete notion of time, with each time corresponding to a *step* taken by the system

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1. Build a formal model of this state-changing system describing
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Example, Vending machine

A state-changing system: vending machine dispensing drinks

- The machine has several components, including:
  - storage space for storing and preparing drinks,
  - a box for dispensing drinks, and
  - a coin slot
- When the machine is operating, it goes through several states, depending on the behavior of the current customer
- Each action by the customer or the machine itself may change its state

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State transition: action that may change the machine's state

Modeling state-changing systems

To build a formal model of a particular state-changing system, we specify its behavior in terms of

1. its state variables
2. the possible values for the state variables
3. the state transitions and how they *change* the values of the state variables

A state can be identified with

- the set of pairs (*variable*, *value*), or
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Transition systems

A *transition system* is a tuple $\mathbb{S} = (S, In, T, \mathcal{X}, dom, L)$, where

1. S is a finite non-empty set, the set of *states* of $\mathbb{S}$
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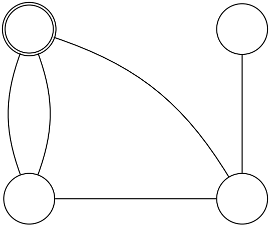
State Transition Graph of a transition system $\mathbb{S} = (S, In, T, \mathcal{X}, dom, L)$:

- The nodes are the states of $\mathbb{S}$
- The arcs are elements of the transition relation:
  there is an edge from state s to state s' iff $(s, s') \in T$



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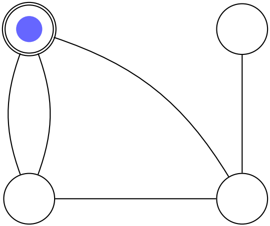

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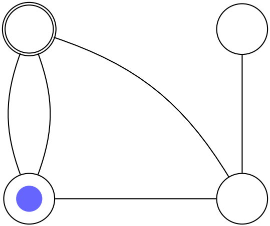

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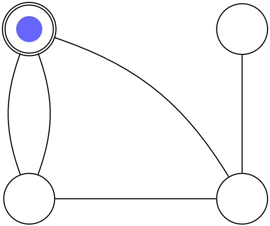

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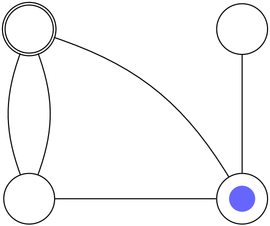

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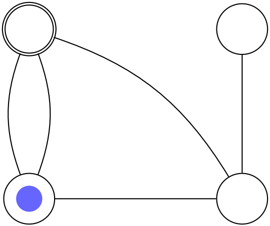

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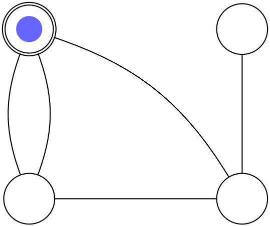

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4. $\mathcal{X}$ is a finite set of *state variables*
5. dom is a mapping from $\mathcal{X}$ such that
   for every $x \in \mathcal{X}$, $dom(x)$ is a non-empty set of values, the *domain of x*
6. L is a function mapping states of $\mathbb{S}$ to interpretations, the *labeling function* of $\mathbb{S}$
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We will only study finite-state transition systems

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Note this part of the definition:

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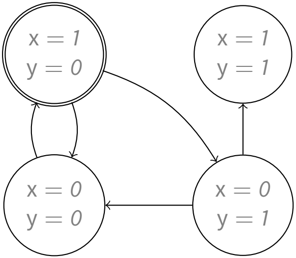
Example: $\mathcal{X} = \{x, y\}$, $dom(x) = dom(y) = \{0, 1\}$



$$
\begin{aligned}
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In &= \{(1,0)\} \\
T &= \{((1,0),(0,1)), \\
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States as interpretations

If $L(s)(x) = v$, we say that x *has the value* v *in* s, and write $s(x) = v$

If $L(s) \models A$, we say that s *satisfies* A *or* A *is true in* s, and write $s \models A$

In both cases, we identify s with $L(s)$

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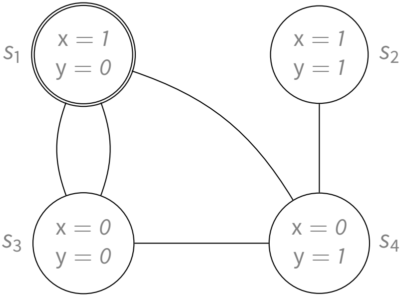

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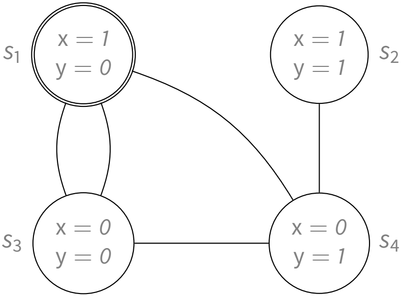

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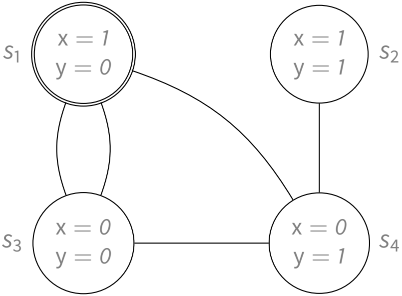

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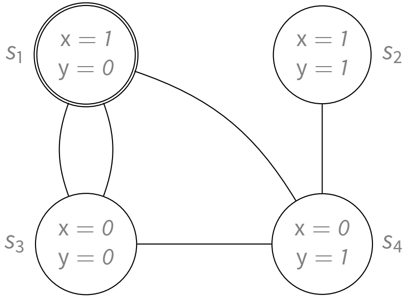

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Transitions

We will usually represent a transition relation as a union of *transitions*

Transition t: any set of state pairs

A transition t is *applicable* to a state s if there is a state s' such that $(s, s') \in t$

A transition t is *deterministic* if for every state s there is at most one state s' such that $(s, s') \in t$

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Vending machine

1. The vending machine contains a drink storage, a coin slot, and a drink dispenser. The drink storage stores drinks of two kinds: beer and coffee. We are only interested in whether a particular kind of drink is currently being stored or not, but not interested in the amount of it.
2. The coin slot can accommodate up to three coins.
3. The drink dispenser can store at most one drink. If it contains a drink, this drink should be removed before the next one can be dispensed.
4. A can of beer costs two coins. A cup of coffee costs one coin.
5. There are two kinds of customers: students and professors. Students drink only beer, professors drink only coffee.
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1. The vending machine contains a drink storage, a coin slot, and a drink dispenser. The drink storage stores drinks of two kinds: beer and coffee. We are only interested in whether a particular kind of drink is currently being stored or not, but not interested in the amount of it.
2. The coin slot can accommodate up to three coins.
3. The drink dispenser can store at most one drink. If it contains a drink, this drink should be removed before the next one can be dispensed.
4. A can of beer costs two coins. A cup of coffee costs one coin.
5. There are two kinds of customers: students and professors. Students drink only beer, professors drink only coffee.
6. From time to time the drink storage can be recharged.

Formalization: Variables and Domains

variable	domain	explanation
st_coffee	$\{0, 1\}$	drink storage contains coffee
st_beer	$\{0, 1\}$	drink storage contains beer
disp	$\{\mathit{none}, \mathit{beer}, \mathit{coffee}\}$	content of drink dispenser
coins	$\{\mathit{0}, \mathit{1}, \mathit{2}, \mathit{3}\}$	number of coins in the slot
customer	$\{\mathit{none}, \mathit{student}, \mathit{prof}\}$	customer

Transitions for the Vending Machine

1. *Recharge*, results in the drink storage having both beer and coffee
2. *Customer_arrives*, corresponds to a customer arriving at the machine
3. *Customer_leaves*, corresponds to the customer's leaving
4. *Coin_insert*, corresponds to the customer's inserting a coin in the machine
5. *Dispense_beer*, results in the customer's getting a can of beer
6. *Dispense_coffee*, results in the customer's getting a cup of coffee
7. *Take_drink*, corresponds to the customer's removing a drink from the dispenser

Symbolic Representation of Sets of States

Let $\mathbb{S} = (S, In, T, \mathcal{X}, dom, L)$ be a (finite-state) labelled transition system

Every PLFD formula F over the variables in $\mathcal{X}$ defines a set states:

$$\{ s \mid s \models F \}$$

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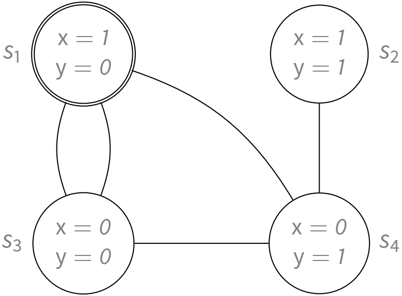
We say that F *(symbolically) represents* this set of states

Symbolic Representation of Sets of States



- $x \leftrightarrow y$
- $x \wedge y$
- $\neg x$

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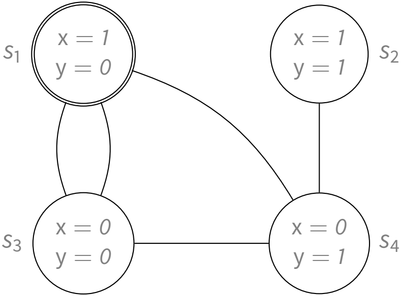

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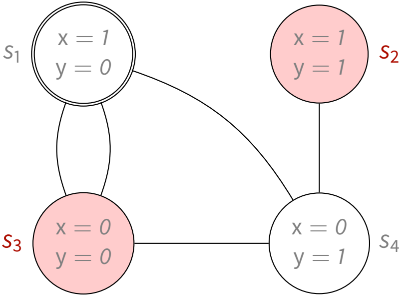

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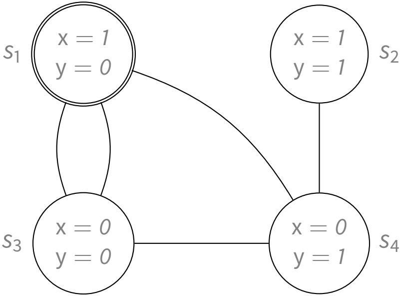

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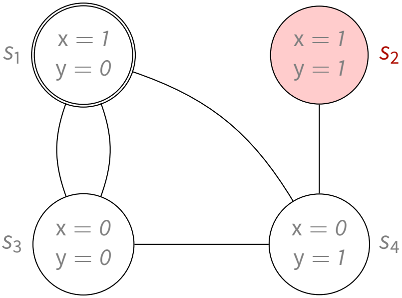

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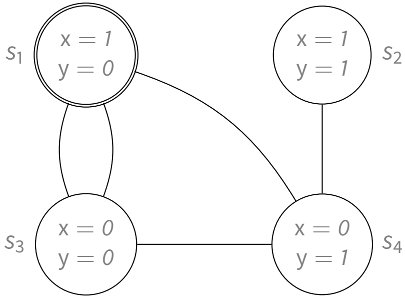

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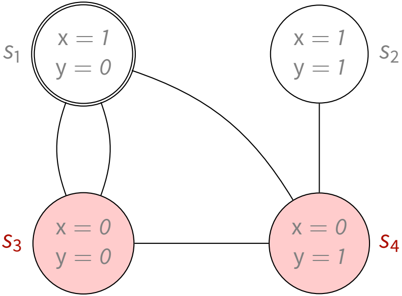

- $x \leftrightarrow y$ represents $\{ s_2, s_3 \}$
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- $\neg x$ represents $\{ s_3, s_4 \}$

Example

Let us represent the set of states in which the machine is ready to dispense a drink.

In every such state, it must be the case that

- a drink is available
- the drink dispenser is empty, and
- the coin slot contains enough coins

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This can be expressed by:

$$
\begin{aligned}
&(\text{st\_coffee} \lor \text{st\_beer}) \land \\
&\text{disp} = \textit{none} \land \\
&((\text{coins} = \textit{1} \land \text{st\_coffee}) \lor \text{coins} = \textit{2} \lor \text{coins} = \textit{3})
\end{aligned}
$$

Symbolic Representation of Transitions

$\mathbb{S} = (S, In, T, \mathcal{X}, dom, L)$

A *transition* t in $\mathbb{S}$ is a binary relation on s, i.e., a set of state pairs:

$$t = \{\, (s, s') \mid s, s' \in S \,\}$$

It takes the system from some *current state* or *pre-state* s
to some *next state* or *post-state* s'

Can we represent transitions symbolically using PLFD formulas?

Not immediately.

PLFD formulas over $\mathcal{X}$ can only express properties of a single state

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How can we represent transitions using formulas?

- Introduce a new set of *next-state variables* $\mathcal{X}' = \{ x' \mid x \in \mathcal{X} \}$
- Treat pairs of states as interpretations of formulas over $\mathcal{X} \cup \mathcal{X}'$

$$\text{For all } x \in \mathcal{X}, \quad (s, s')(x) \stackrel{\text{def}}{=} s(x)$$
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The transition *Recharge*:

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However, this formula describes a strange transition after which, for example

- coins may appear in and disappear from the slot
- drinks may appear in and disappear from the dispenser
- ...

Frame problem

We must express explicitly, possibly for a large number of state variables, that

the values of these variables do not change after a transition

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\begin{array}{l}
(\text{coins} = 0 \leftrightarrow \text{coins}' = 0) \ \wedge \\
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This is known as the *frame problem*

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The frame formula

$$\mathbb{S} = (S, In, T, \mathcal{X}, dom, L)$$

Notation:

When $dom(x) = dom(y)$,

$$x \neq v \quad \stackrel{\text{def}}{=} \quad \neg(x = v)$$
$$x = y \quad \stackrel{\text{def}}{=} \quad \bigwedge_{v \in dom(x)} (x = v \leftrightarrow y = v)$$

For $\{ x_1, \ldots, x_n \} \subseteq \mathcal{X}$,

$$only(x_1, \ldots, x_n) \quad \stackrel{\text{def}}{=} \quad \bigwedge_{x \in \mathcal{X} \setminus \{ x_1, \ldots, x_n \}} x = x'$$

$only(x_1, \ldots, x_n)$ can be used in symbolic transitions to state that $x_1, \ldots, x_n$ are the only variables whose values may change in the transition

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Preconditions and postconditions

$\mathbb{S} = (S, In, T, \mathcal{X}, dom, L)$

Typical symbolic representation of a transition t in $\mathbb{S}$:

A PLFD formula $F_1 \wedge F_2$ where

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precondition: a necessary condition for a state of $\mathbb{S}$ of to be a pre-state of t

postcondition: a condition relating t's post-states to their corresponding pre-state

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1. *Recharge*, results in the drink storage having both beer and coffee
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Transitions: Symbolic Representation

Recharge
Customer_arrives
Customer_leaves
Coin_insert

precondition *postcondition*

$$\mathit{Recharge} \stackrel{\text{def}}{=} \text{customer} = \mathit{none} \wedge \text{st\_coffee}' \wedge \text{st\_beer}' \wedge \mathit{only}(\text{st\_coffee}, \text{st\_beer})$$

$$\mathit{Customer\_arrives} \stackrel{\text{def}}{=} \text{customer} = \mathit{none} \wedge \text{customer}' \neq \mathit{none} \wedge \mathit{only}(\text{customer})$$

$$\mathit{Customer\_leaves} \stackrel{\text{def}}{=} \text{customer} \neq \mathit{none} \wedge \text{customer}' = \mathit{none} \wedge \mathit{only}(\text{customer})$$

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Transitions: Symbolic Representation

Recharge
Customer_arrives
Customer_leaves
Coin_insert

$$
\begin{array}{rcll}
 & & \textit{precondition} & \textit{postcondition} \\
\textit{Recharge} & \overset{\text{def}}{=} & \text{customer} = \textit{none} & \wedge\ \text{st\_coffee}' \wedge \text{st\_beer}' \\
 & & & \wedge\ \textit{only}(\text{st\_coffee}, \text{st\_beer}) \\
\textit{Customer\_arrives} & \overset{\text{def}}{=} & \text{customer} = \textit{none} & \wedge\ \text{customer}' \neq \textit{none} \\
 & & & \wedge\ \textit{only}(\text{customer}) \\
\textit{Customer\_leaves} & \overset{\text{def}}{=} & \text{customer} \neq \textit{none} & \wedge\ \text{customer}' = \textit{none} \\
 & & & \wedge\ \textit{only}(\text{customer}) \\
\textit{Coin\_insert} & \overset{\text{def}}{=} & \text{customer} \neq \textit{none} & \wedge\ (\text{coins} = \mathit{0} \rightarrow \text{coins}' = \mathit{1}) \\
 & \wedge & \text{coins} \neq \mathit{3} & \wedge\ (\text{coins} = \mathit{1} \rightarrow \text{coins}' = \mathit{2}) \\
 & & & \wedge\ (\text{coins} = \mathit{2} \rightarrow \text{coins}' = \mathit{3}) \\
 & & & \wedge\ \textit{only}(\text{coins})
\end{array}
$$

Transitions: Symbolic Representation

Recharge
Customer_arrives
Customer_leaves
Coin_insert

precondition — *postcondition*

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Transitions

Dispense_beer
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$$
\begin{aligned}
\textit{Dispense\_beer} \stackrel{\text{def}}{=}\ & \text{customer} = \textit{student} \wedge \text{st\_beer} \wedge \\
& \text{disp} = \textit{none} \wedge (\text{coins} = \textit{2} \vee \text{coins} = \textit{3}) \wedge \\
& (\text{coins} = \textit{2} \rightarrow \text{coins}' = \textit{0}) \wedge \\
& (\text{coins} = \textit{3} \rightarrow \text{coins}' = \textit{1}) \wedge \\
& \text{disp}' = \textit{beer} \wedge \textit{only}(\text{st\_beer}, \text{disp}, \text{coins})
\end{aligned}
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$$

Transitions

Dispense_beer
Dispense_coffee
Take_drink

$$\mathit{Dispense\_beer} \stackrel{\text{def}}{=} \begin{array}{l} \text{customer} = \mathit{student} \land \text{st\_beer} \land \\ \text{disp} = \mathit{none} \land (\text{coins} = \mathit{2} \lor \text{coins} = \mathit{3}) \land \\ (\text{coins} = \mathit{2} \rightarrow \text{coins}' = \mathit{0}) \land \\ (\text{coins} = \mathit{3} \rightarrow \text{coins}' = \mathit{1}) \land \\ \text{disp}' = \mathit{beer} \land \mathit{only}(\text{st\_beer}, \text{disp}, \text{coins}) \end{array}$$

$$\mathit{Dispense\_coffee} \stackrel{\text{def}}{=} \begin{array}{l} \text{customer} = \mathit{prof} \land \text{st\_coffee} \land \\ \text{disp} = \mathit{none} \land \text{coins} \neq \mathit{0} \land \\ (\text{coins} = \mathit{1} \rightarrow \text{coins}' = \mathit{0}) \land \\ (\text{coins} = \mathit{2} \rightarrow \text{coins}' = \mathit{1}) \land \\ (\text{coins} = \mathit{3} \rightarrow \text{coins}' = \mathit{2}) \land \\ \text{disp}' = \mathit{coffee} \land \mathit{only}(\text{st\_coffee}, \text{disp}, \text{coins}) \end{array}$$

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$$\begin{aligned}
\textit{Dispense_beer} \stackrel{\text{def}}{=}\ & \text{customer} = \textit{student} \wedge \text{st_beer} \wedge \\
& \text{disp} = \textit{none} \wedge (\text{coins} = \textit{2} \vee \text{coins} = \textit{3}) \wedge \\
& (\text{coins} = \textit{2} \rightarrow \text{coins}' = \textit{0}) \wedge \\
& (\text{coins} = \textit{3} \rightarrow \text{coins}' = \textit{1}) \wedge \\
& \text{disp}' = \textit{beer} \wedge \textit{only}(\text{st_beer}, \text{disp}, \text{coins}) \\
\textit{Dispense_coffee} \stackrel{\text{def}}{=}\ & \text{customer} = \textit{prof} \wedge \text{st_coffee} \wedge \\
& \text{disp} = \textit{none} \wedge \text{coins} \neq \textit{0} \wedge \\
& (\text{coins} = \textit{1} \rightarrow \text{coins}' = \textit{0}) \wedge \\
& (\text{coins} = \textit{2} \rightarrow \text{coins}' = \textit{1}) \wedge \\
& (\text{coins} = \textit{3} \rightarrow \text{coins}' = \textit{2}) \wedge \\
& \text{disp}' = \textit{coffee} \wedge \textit{only}(\text{st_coffee}, \text{disp}, \text{coins}) \\
\textit{Take_drink} \stackrel{\text{def}}{=}\ & \text{customer} \neq \textit{none} \wedge \text{disp} \neq \textit{none} \wedge \\
& \text{disp}' = \textit{none} \wedge \textit{only}(\text{disp})
\end{aligned}$$

Temporal properties of transition systems

1. There is no state in which professor and student are both customers.
2. Students always get beer.
3. The machine cannot dispense drinks forever without recharging.
4. Eventually, the machine runs out of beer.
5. If coffee is dispensed the machine must have had coins right before.
6. If the machine is never recharged it will never dispense drinks.
7. …

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