

CS:5810

Formal Methods in Software Engineering

Introduction to Alloy

Part 1

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Outline

- Introduction to basic Alloy constructs using a simple example of a static model
 - How to define **sets, subsets, relations with multiplicity constraints**
 - How to use Alloy's **quantifiers** and **predicate** forms
- Basic use of the Alloy Analyzer 4 (AA)
 - **Loading, compiling, and analyzing** a simple Alloy specification
 - Adjusting basic **tool parameters**
 - Using the **visualization** tool to view instances of models

Roadmap

Alloy: Rationale and Use Strategies

- What types of systems have been modeled with Alloy
- What types of questions can AA answer
- What is the purpose of each of the sections in an Alloy specification

Alloy Specifications

- Parameterized conditionals
- Indexed relations
- Graphical representations of Alloy models
- More complex examples

Alloy --- Why was it created?

Lightweight

small and easy to use, and capable of expressing common properties tersely and naturally

Precise

having a simple and uniform mathematical semantics

Tractable

amenable to efficient and fully automated semantic analysis (within scope limits)

Alloy --- Comparison

UML

- Has similarities (graphical notation, OCL constraints) but it is neither lightweight, nor precise
- UML includes many modeling notions omitted from Alloy (use-cases, state-charts, code architecture specs)
- Alloy's diagrams and relational navigation are inspired by UML

Z

- Precise, but intractable. Stylized typography makes it harder to work with.
- Z is more expressive than Alloy, but more complicated
- Alloy's set-based semantics is inspired by Z

Alloy --- What is it used for?

Alloy is a textual modeling language aimed at expressing **structural** and **behavioral** properties of software systems

It is not meant for modeling code architecture (*a la* class diagrams in UML)

But there might be a close relationship between the Alloy specification and an implementation in an OO language

Alloy --- Example Applications

The Alloy 4 distribution comes with several example models that together illustrate the use of Alloy's constructs

Examples

- Specification of a distributed spanning tree
- Model of a generic file system
- Model of a generic file synchronizer
- Tower of Hanoi model
- ...

Alloy in General

Alloy is general enough that it can model

- any domain of individuals and
- relations between them

We will then start with a few simple examples that are not necessarily about software

Example: Family Structure

We want to...

- Model **parent/child relationships** as primitive relations
- Model **spousal relationships** as primitive relations
- Model relationships such as “**siblings**” as *derived* relations
- Enforce certain **biological constraints** via 1st-order predicates (e.g., people have only one mother)
- Enforce certain **social constraints** via 1st-order predicates (e.g., a wife isn't a sibling)
- Confirm or refute the existence of certain **derived relationships** (e.g., no one has a wife with whom they share a parent)

Example: addressBook

An **address book** for an email client that maintains a mapping from **names** to **addresses**

FriendBook
Ted -> ted@gmail.com Ryan -> ryan@hotmail.com

WorkBook
Pilard -> lpilard@uiowa.edu Ryan -> ryan@uiowa.edu

Atoms and Relations

In Alloy, everything is built from **atoms** and **relations**

An **atom** is a primitive entity that is

- *indivisible*: it cannot be broken down into smaller parts
- *immutable*: its properties do not change over time
- *uninterpreted*: it does not have any built in property
(the way numbers do for example)

A **relation** is a structure that **relates atoms**. It is a set of **tuples**, each tuple being a sequence of atoms

Atoms and Relations: Examples

- **Unary relations**: a set of names, a set of addresses and a set of books

Name = {(N0), (N1), (N2)} ← Atoms

Addr = {(D0), (D1)} ← Atoms

Book = {(B0), (B1)} ← Atoms

- A **binary relation** from names to addresses

address = {(N0, D0), (N1, D1)} ← Tuples

- A **ternary relation** from books to name to addresses

addr = {(B0, N0, D0), (B0, N1, D1), (B1, N1, D2)} ← Tuples

Relations

Size of a relation: the number of tuples in the relation

Arity of a relation: the number of atoms in each tuple of the relation

relations with arity 1, 2, and 3 are said to be *unary*, *binary*, and *ternary* relations

Examples.

- relation of arity 1 and size 1: `myName = {(N0)}`
- relation of arity 2 and size 3: `address = {(N0,D0),(N1,D1),(N2,D1)}`

Main Components of Alloy Models

- Signatures and Fields
- Predicates and Functions
- Facts
- Assertions
- Commands and scopes

Signatures and Fields

Signatures

- Describe classes of entities we want to reason about
- Sets defined in signatures are fixed (dynamic aspects can be modeled by time-dependent relations)

Fields

- Define relations between signatures

Simple constraints

- Multiplicities on signatures
- Multiplicities on relations

Signatures

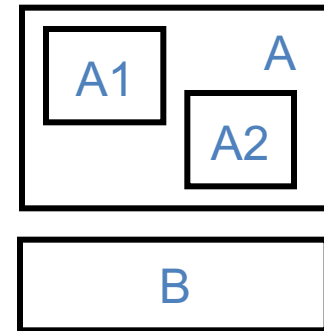
- A **signature** introduces a set of **atoms**
- The declaration
`sig A {}`
introduces a set named **A**
- A set can be introduced as an **extension** of another;
thus

`sig A1 extends A {}`

introduces a set **A1** that is a **subset** of **A**

Signatures

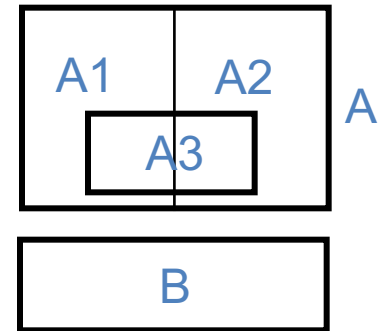
```
sig A {}  
sig B {}  
sig A1 extends A {}  
sig A2 extends A {}
```



- A1 and A2 are **extensions** of A
- A signature declared independently of any other one is a **top-level signature**, e.g., A and B
- Extensions of the same signature are **mutually disjoint**, as are top-level signatures

Signatures

```
abstract sig A {}  
sig B {}  
sig A1 extends A {}  
sig A2 extends A {}
```



- A signature can be introduced as a **subset** of another
`sig A3 in A {}`
- An **abstract signature** has no elements except those belonging to its extensions or subsets
- All extensions of an abstract signature `A` form a **partition** of `A`

Fields

- **Relations** are declared as **fields** of signatures

- Writing

`sig A {f: e}`

introduces a relation `f` of type `A x e`, where `e` is an expression denoting a product of signatures)

- Examples: (with signatures `A`, `B`, `C`)

- Binary Relation:

`sig A { f1: B }` // `f1` is a subset of `A x B`

- Ternary Relation:

`sig A { f2: B -> C }` // `f2` is a subset of `A x B x C`

Example Signatures and Fields

Family Structure:

```
abstract sig Person {  
    children: Person,  
    siblings: Person  
}
```

```
sig Man, Woman extends Person {}
```

```
sig Married in Person {  
    spouse: Married  
}
```

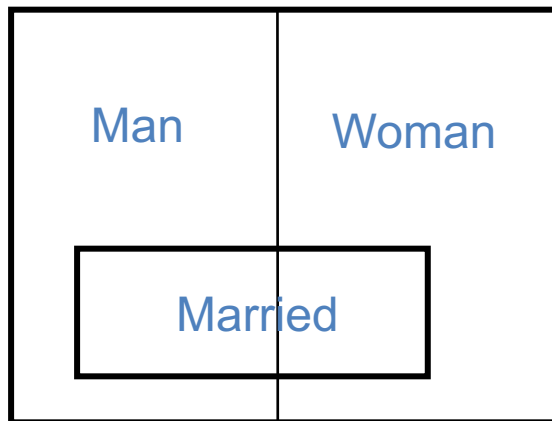
Fields



Example: Family Structure

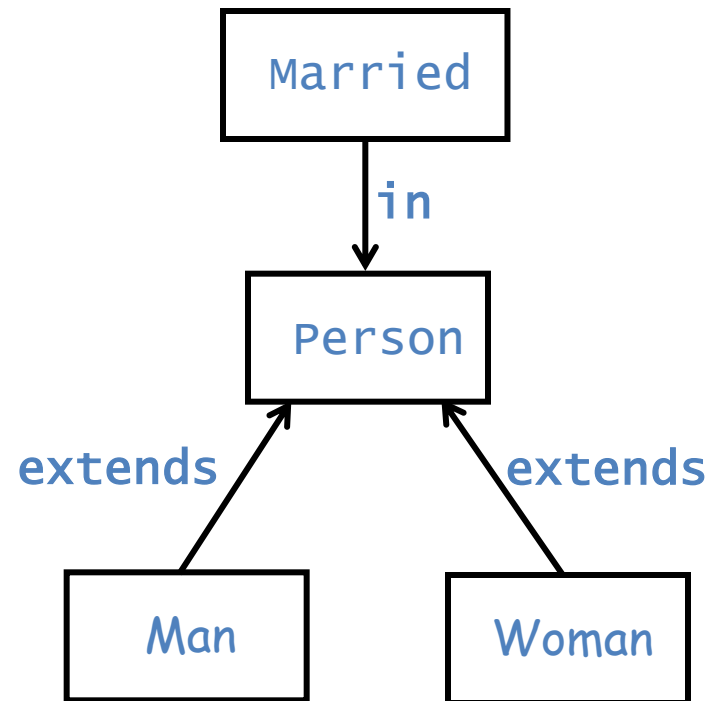
Alloy Model

```
abstract sig Person {}  
sig Man extends Person {}  
sig Woman extends Person {}  
sig Married in Person {}
```



Person

Graphical Representation



Model Instances

The Alloy Analyzer will generate **instances** of models so that we can check if they match our intentions.

Which of the following are instances of our current model?

```
abstract sig Person {}
sig Man extends Person {}
sig Woman extends Person {}
sig Married in Person {}
```

~~1.~~ Person = {(P0),(P1),(P2)}
Man = {(P1),(P2)}
Married = {}
Woman = {(P0),(P1)}

~~2.~~ Person = {(P0),(P1)}
Man = {(P0)}
Married = {(P1)}
Woman = {}

✓ 3. Person = {(P0),(P1),(P2)}
Man = {(P1),(P2)}
Married = {}
Woman = {(P0)}

✓ 4. Person = {(P0),(P1),(P2),(P3)}
Man = {(P0),(P1),(P2),(P3)}
Married = {(P2),(P3)}
Woman = {}

✓ 5. Person = {(P0),(P1)}
Man = {(P0)}
Married = {(P1),(P0)}
Woman = {(P1)}

Example: Family Structure

Alloy Model with siblings

```
abstract sig Person {  
  siblings: Person  
}  
sig Man extends Person {}  
sig Woman extends Person {}  
sig Married in Person {}
```

siblings is a binary relation
it is a subset of Person x Person

Example instance

```
Person = {(P0), (P1)}  
Man = {(P0), (P1)}  
Married = {}  
Woman = {}
```

```
siblings = {(P0,P1), (P1,P0)}
```

Intuition: P0 and P1 are siblings

Multiplicities

Allow us to constrain the **sizes** of sets

- A multiplicity keyword placed before a signature declaration constrains the number of element in the signature's set

```
m sig A {}
```

- We can also make multiplicities constraints on fields

```
sig A {f: m e}
```

```
sig A {f: e1 m -> n e2}
```

There are four multiplicities:

- **set** : any number
- **some** : one or more
- **lone** : zero or one
- **one** : exactly one

Multiplicities: Examples

Without multiplicity:

A set of colors, each of which is red, yellow or green

```
abstract sig Color {}  
sig Red, Yellow, Green extends Color {}
```

With multiplicity:

An enumeration of colors

```
abstract sig Color {}  
one sig Red, Yellow, Green extends Color {}
```

Multiplicities: Examples

- A file system in which each directory contains any number of objects, and each alias points to exactly one object

```
abstract sig Object {}  
sig Directory extends Object {contents: set Object}  
sig File extends Object {}  
sig Alias in File {to: one Object}
```

- The **default multiplicity** for fields is **one**, so:

```
sig A {f: e}    and    sig A {f: one e}
```

are equivalent

redundant

Multiplicities: Examples

- A book maps names to addresses
 - There is at most one address per Name
 - An address is associated to at least one name

```
sig Name, Addr {}
```

```
sig Book {
```

```
  addr: Name some -> 1one Addr
```

```
}
```

Multiplicities: Examples

- A collection of weather forecasts, each of which has a field `weather` associating every city with exactly one weather condition

```
sig Forecast {weather: City -> one weather}
```

```
sig City {}
```

```
abstract sig weather {}
```

```
one sig Rainy, Sunny, Cloudy extends weather {}
```

- Instance:

```
City = {(Iowa City), (Chicago)}
```

```
Rainy = {(rainy)}
```

```
Sunny = {(sunny)}
```

```
Cloudy = {(cloudy)}
```

```
Forecast = {(f1), (f2)}
```

```
weather = { (f1, Iowa City, rainy), (f1, Chicago, rainy),  
            (f2, Iowa City, rainy), (f2, Chicago, sunny) }
```

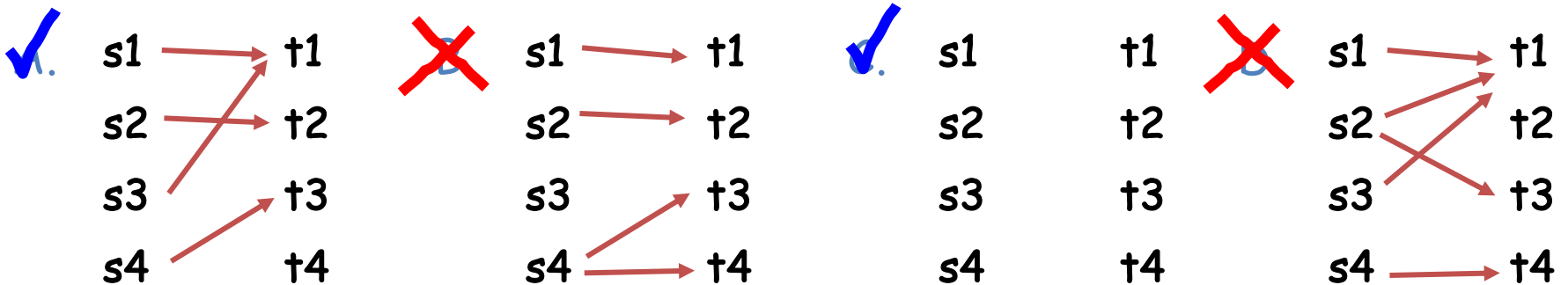
Multiplicities and Binary Relations

- $\text{sig } S \{ f: \text{!one } T \}$

- says that, for each element s of S , f maps s to **at most one** value in T

Conventional name: **partial function**

- Potential instances:



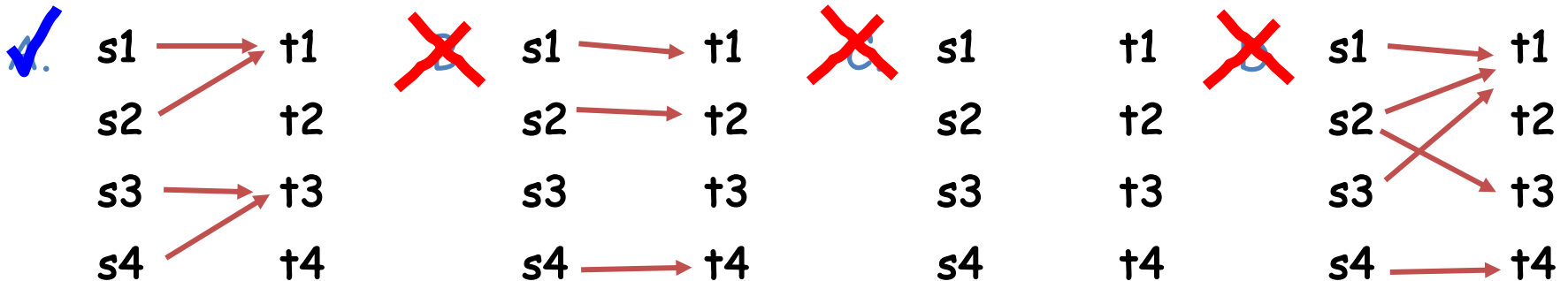
Multiplicities and Binary Relations

- $\text{sig } S \{f: \text{one } T\}$

- says that, for each element s of S , f maps s to **exactly one** value in T

Conventional name: total function

- Potential instances:

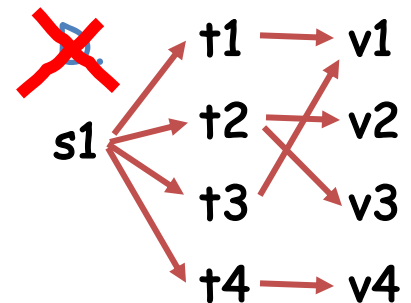
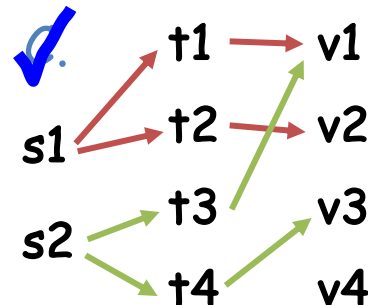
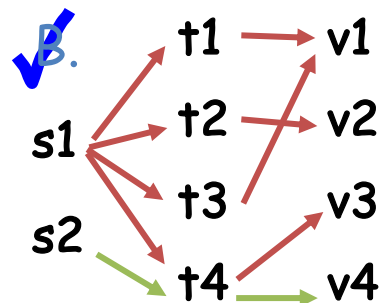
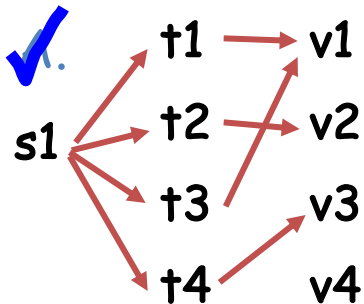


Multiplicities and Ternary Relations

- **sig** $S \{f: T \rightarrow \text{one } V\}$

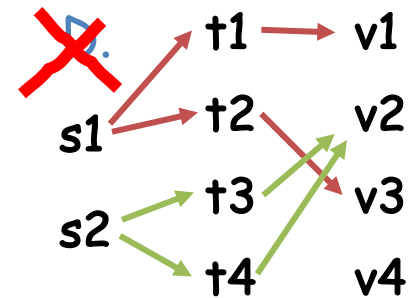
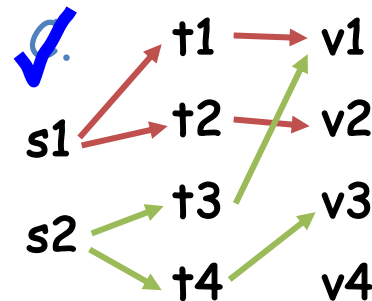
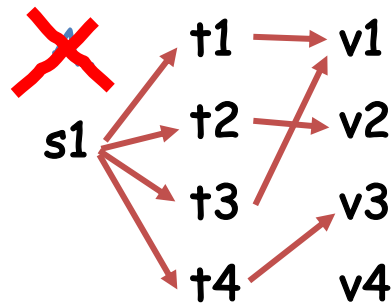
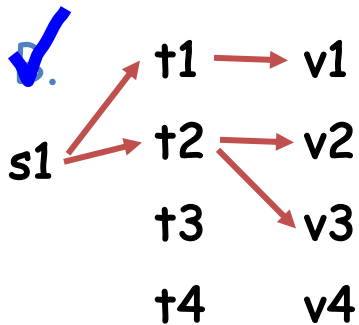
- For **each element** s of S , over the triples that start with s :
 f maps each T -element to **exactly one** V -element

- Potential instances:



Multiplicities and Ternary Relations

- $\text{sig } S \{f: T \text{ } \lceil \text{one} \rightarrow V\}$
 - For each element s of S , over the triples that start with s :
 f maps at most one T -element to the same V -element
- Potential instances:



Multiplicities and Relations

- Other kinds of relational structures can be specified using multiplicities
- Examples:
 - `sig S {f: some T} ... total relation`
 - `sig S {f: set T} ... partial relation`
 - `sig S {f: T set -> set V}`
 - `sig S {f: T one -> V}`
 - ...

Cardinality Constraints

Multiplicities can also be applied to whole expressions denoting relations

- **some** e e is non-empty
- **no** e e is empty
- **1one** e e has at most one tuple
- **one** e e has exactly one tuple

Example: Family Structure

- How would you use multiplicities to define the **children** relation?

```
sig Person { children: set Person }
```

– Intuition: each person has zero or more children

- How would you use multiplicities to define the **spouse** relation?

```
sig Married { spouse: one Married }
```

– Intuition: each married person has exactly one spouse

Summarizing

Alloy Model

```
abstract sig Person {  
    children: set Person,  
    siblings: set Person  
}
```

```
sig Man, Woman extends Person {}
```

```
sig Married in Person {  
    spouse: one Married  
}
```

Exercises

- Start the Alloy Analyzer
- Load file `family-1.a1s` from the **Resources** section of the course website
- Execute it
- Analyze the model instance
- Look at the generated instance
- Does it look correct?
- What, if anything, would you change about it?

Model Instance

Instance found:

```
Person = {Man0,Man1,Man2}
Man = {Man0,Man1,Man2}
Woman = {}
Married = {Man0,Man1,Man2}
```

```
children = { (Man0,Man0), (Man0,Man1),
             (Man1,Man0),
             (Man2,Man1), (Man2,Man2)
           }
```

```
siblings = { (Man0,Man0), (Man0,Man1),
             (Man1,Man0), (Man1,Man2),
             (Man2,Man2)
           }
```

```
spouse = { (Man1,Man0), (Man0,Man2), (Man2,Man0) }
```

```
abstract sig Person {
    children: set Person,
    siblings: set Person }
sig Man, Woman extends Person {}
sig Married in Person {
    spouse: one Married }
```

No Women?

Instance found:

```
Person = {Man0, Man1, Man2}
Man = {Man0, Man1, Man2}
Woman = {}
Married = {Man0, Man1, Man2}
```

```
children = { (Man0, Man0), (Man0, Man1),
             (Man1, Man0),
             (Man2, Man1), (Man2, Man2)
           }
```

```
siblings = { (Man0, Man0), (Man0, Man1),
             (Man1, Man0), (Man1, Man2),
             (Man2, Man2)
           }
```

```
spouse = { (Man1, Man0), (Man0, Man2), (Man2, Man0) }
```

```
abstract sig Person {
    children: set Person,
    siblings: set Person }
sig Man, Woman extends Person {}
sig Married in Person {
    spouse: one Married }
```

Man can be his own child ?

Instance found:

```
Person = {Man0,Man1,Man2}
Man = {Man0,Man1,Man2}
Woman = {}
Married = {Man0,Man1,Man2}
```

```
children = { (Man0,Man0), (Man0,Man1),
             (Man1,Man0),
             (Man2,Man1), (Man2,Man2)
           }
siblings = { (Man0,Man0), (Man0,Man1),
             (Man1,Man0), (Man1,Man2),
             (Man2,Man2)
           }
spouse = { (Man1,Man0), (Man0,Man2), (Man2,Man0) }
```

```
abstract sig Person {
    children: set Person,
    siblings: set Person }
sig Man, Woman extends Person {}
sig Married in Person {
    spouse: one Married }
```


Multiple Fathers?

Instance found:

```
Person = {Man0, Man1, Man2}
Man = {Man0, Man1, Man2}
Woman = {}
Married = {Man0, Man1, Man2}
```

```
children = { (Man0, Man0), (Man0, Man1),
             (Man1, Man0),
             (Man2, Man1), (Man2, Man2)
           }
```

```
siblings = { (Man0, Man0), (Man0, Man1),
             (Man1, Man0), (Man1, Man2),
             (Man2, Man2)
           }
```

```
spouse = { (Man1, Man0), (Man0, Man2), (Man2, Man0) }
```

```
abstract sig Person {
    children: set Person,
    siblings: set Person }
sig Man, Woman extends Person {}
sig Married in Person {
    spouse: one Married }
```

Own-Siblings?

Instance found:

```
Person = {Man0, Man1, Man2}
Man = {Man0, Man1, Man2}
Woman = {}
Married = {Man0, Man1, Man2}
```

```
children = { (Man0, Man0), (Man0, Man1),
             (Man1, Man0),
             (Man2, Man1), (Man2, Man2)
           }
```

```
siblings = { (Man0, Man0), (Man0, Man1),
             (Man1, Man0), (Man1, Man2),
             (Man2, Man2)
           }
```

```
spouse = { (Man1, Man0), (Man0, Man2), (Man2, Man0) }
```

```
abstract sig Person {
    children: set Person,
    siblings: set Person }
sig Man, Woman extends Person {}
sig Married in Person {
    spouse: one Married }
```

Asymmetric Siblings?

Instance found:

```
Person = {Man0, Man1, Man2}
Man = {Man0, Man1, Man2}
Woman = {}
Married = {Man0, Man1, Man2}
```

```
children = { (Man0, Man0), (Man0, Man1),
             (Man1, Man0),
             (Man2, Man1), (Man2, Man2)
           }
```

```
siblings = { (Man0, Man0), (Man0, Man1),
             (Man1, Man0), (Man1, Man2),
             (Man2, Man2)
           }
```

```
spouse = { (Man1, Man0), (Man0, Man2), (Man2, Man0) }
```

```
abstract sig Person {
    children: set Person,
    siblings: set Person }
sig Man, Woman extends Person {}
sig Married in Person {
    spouse: one Married }
```

No (Man2, Man1)?

Child and Sibling?

Instance found:

```
Person = {Man0, Man1, Man2}
Man = {Man0, Man1, Man2}
Woman = {}
Married = {Man0, Man1, Man2}
```

```
children = { (Man0, Man0), (Man0, Man1),
             (Man1, Man0),
             (Man2, Man1), (Man2, Man2)
           }
siblings = { (Man0, Man0), (Man0, Man1),
             (Man1, Man0), (Man1, Man2),
             (Man2, Man2)
           }
spouse = { (Man1, Man0), (Man0, Man2), (Man2, Man0) }
```

```
abstract sig Person {
    children: set Person,
    siblings: set Person }
sig Man, Woman extends Person {}
sig Married in Person {
    spouse: one Married }
```

Asymmetric Marriage?

Instance found:

```
Person = {Man0, Man1, Man2}
Man = {Man0, Man1, Man2}
Woman = {}
Married = {Man0, Man1, Man2}
```

```
abstract sig Person {
    children: set Person,
    siblings: set Person }
sig Man, Woman extends Person {}
sig Married in Person {
    spouse: one Married }
```

```
children = { (Man0, Man0), (Man0, Man1),
             (Man1, Man0),
             (Man2, Man1), (Man2, Man2)
```

```
}
```

```
siblings = { (Man0, Man0), (Man0, Man1),
             (Man1, Man0), (Man1, Man2),
             (Man2, Man2)
```

```
}
```

```
spouse = { (Man1, Man0), (Man0, Man2), (Man2, Man0) }
```

where is (Man0, Man1)?

Model Weaknesses

- The model is **underconstrained**
 - It doesn't match our domain knowledge
 - We can **add constraints** to enrich the model
- Under-constrained models are common early on in the development process
 - AA gives us quick feedback on weaknesses in our model
 - We can **incrementally add** constraints until we are satisfied with it

Adding Constraints

We'd like to enforce the following constraints which are simply matters of **biology**

- *No persons can be their own parent (or more generally, their own ancestor)*
- *No person can have more than one father or mother*
- *A person's siblings are those with the same parents as the person's parents*

Adding Constraints

- We'd like to enforce the following **social** constraints
 - *The spouse relation is symmetric*
 - *A person's spouse cannot be one of his/her siblings*