# CS4980: Computational Epidemiology

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https://homepage.cs.uiowa.edu/~sriram/4980/spring20/

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Epidemiology is not restricted to the study of contagion, nor should it be confused with immunology (the study of an agent's contagion defense system).

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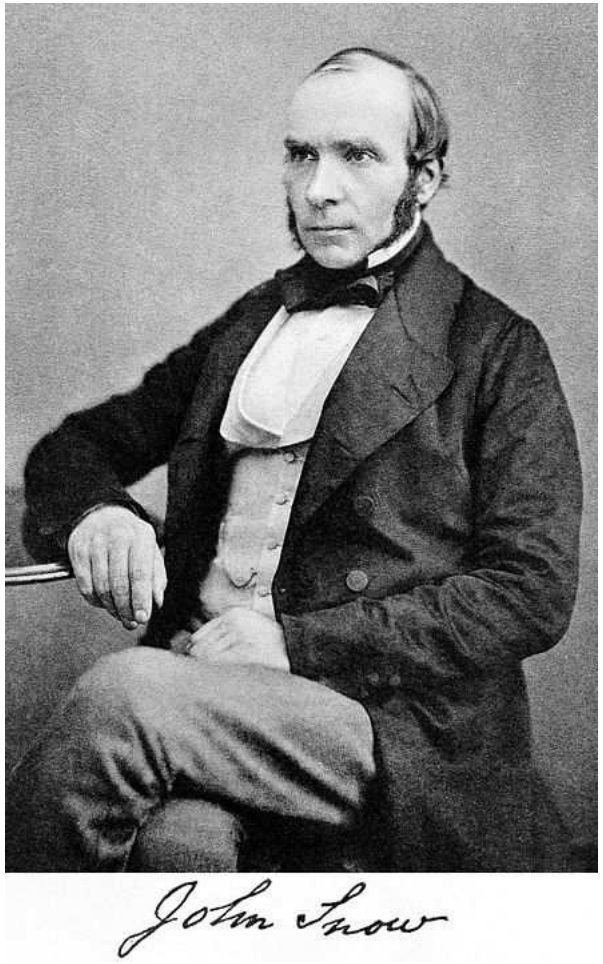
At the time, the primary theory of disease was the *miasma theory*, where breathing “bad air” (Italian: "mal aria") made you sick (and there was plenty of bad air in 1854 London).

# John Snow

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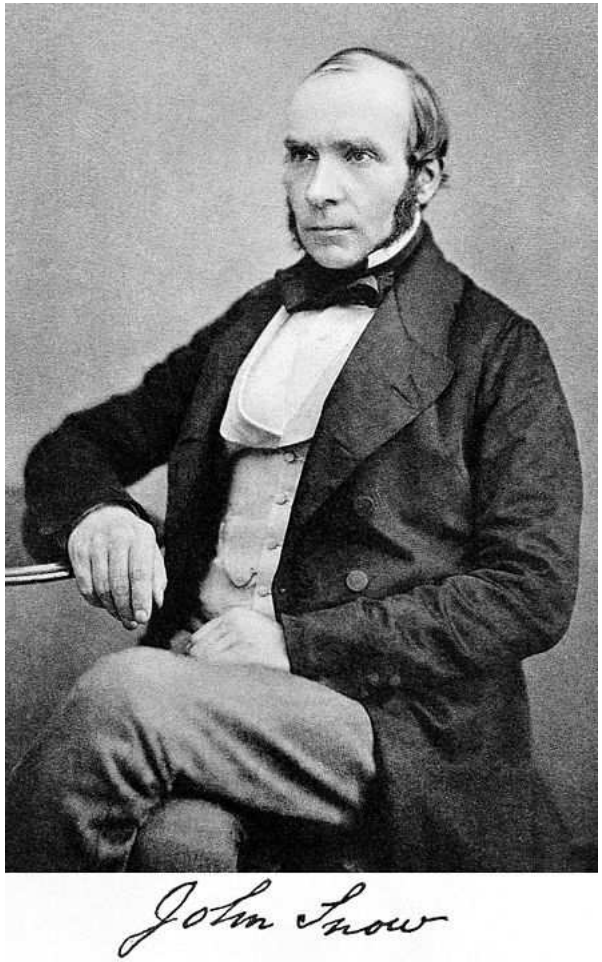


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His analysis of the 1854 cholera outbreak in his neighborhood was published in his 1856 report *On the Mode of the Communication of Cholera*.

https://youtu.be/lNjrAXGRda4

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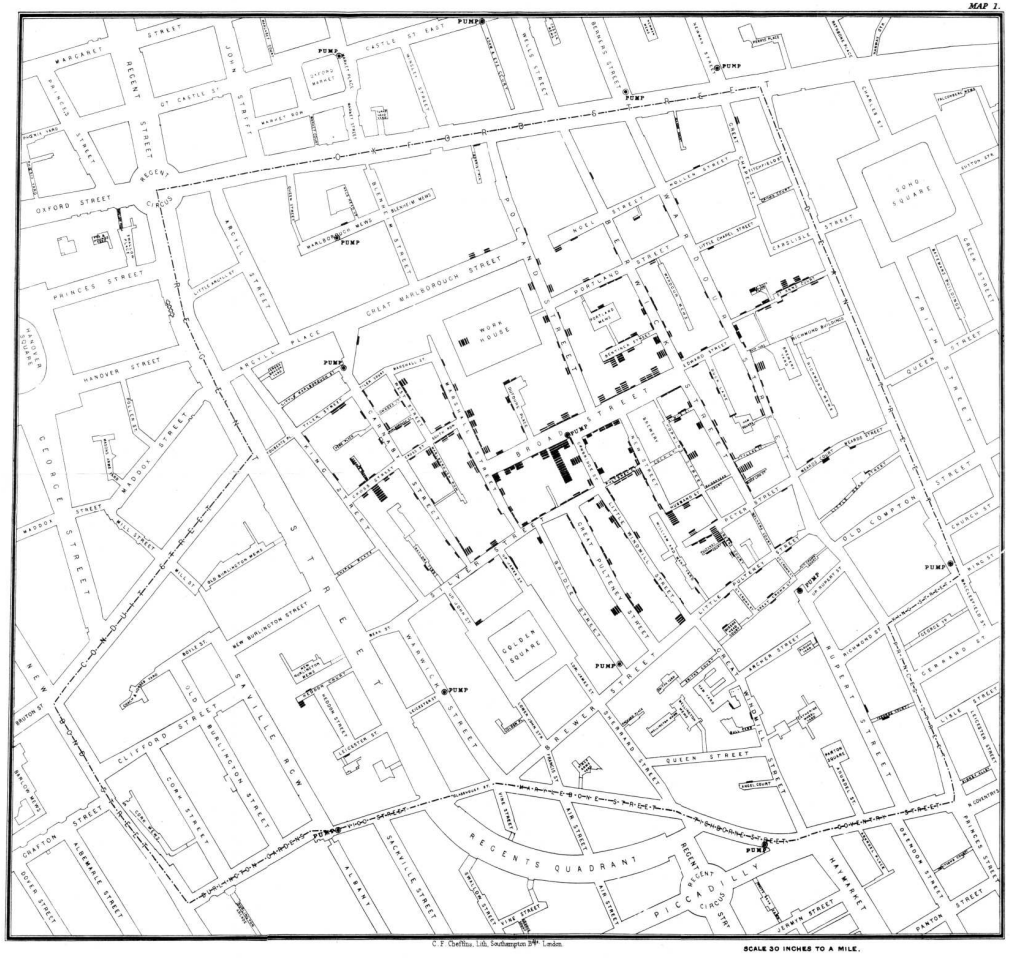


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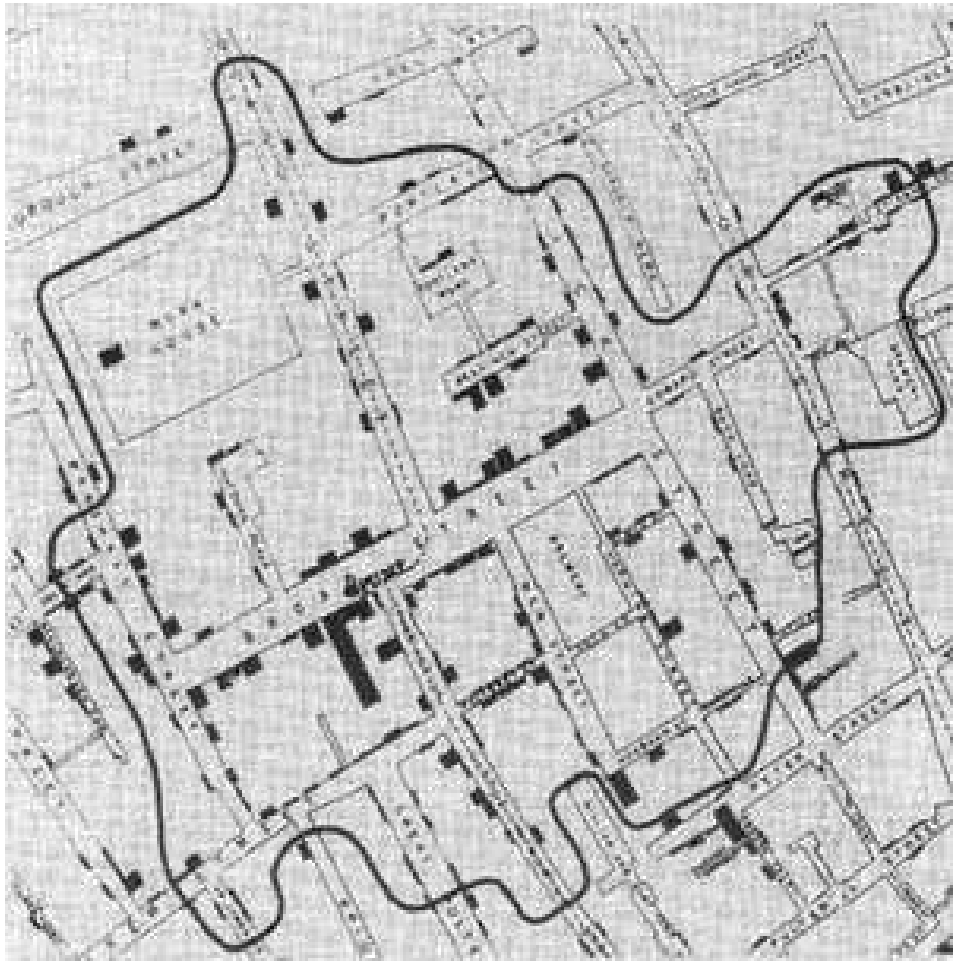
# Voronoi Diagram in "Step Space"



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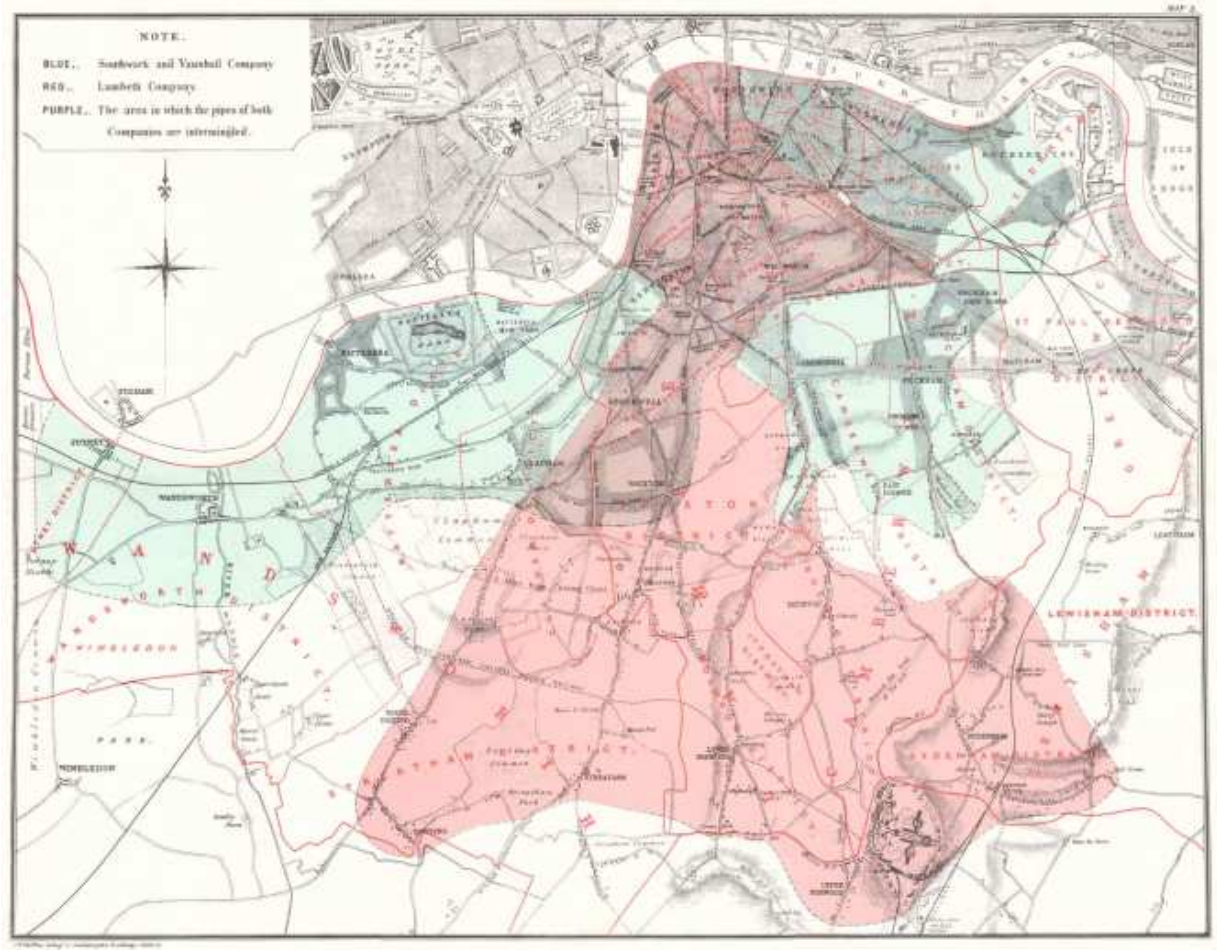
Snow then compared cholera counts among these two very similar populations served by the different companies to support his theory that cholera was water borne.

| Supplier | Number of houses | Cholera deaths | Deaths per 10,000 houses |
|---|---|---|---|
| S&V | 40,046 | 1,263 | 315 |
| Lambeth | 26,107 | 98 | 37 |
| Rest of London | 256,423 | 1,422 | 59 |

# Snow's "Grand Experiment" of 1854



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He was ultimately interested in making recommendations to public officials.

# What is Computational Epidemiology?

"To understand the behavior of complex biological systems, it is useful to devise computer based models by approximating the interactions, via biomathematical expressions. Without doubt, these models could be over simplifications of complex interactions but they would be useful in comparison to classical laboratory experimental approaches which may not be practical or feasible." [Habtemariam *et al.*, 1988]

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Simulation is just one of the "new ideas" that distinguish computational epidemiology from traditional epidemiology.

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Of course, the value of a simulation is limited by the quality of the underlying model and the values of any necessary parameters.

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Note: *Monte Carlo simulations* use randomness to estimate the solution of a mathematical model: here, randomness of the algorithm is not a feature of the model itself. The original post-war simulation is now considered a calculational tool, and not really a "simulation."

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We'll also talk about surveillance and interventions; how does one detect the presence of disease? How does one control its spread, and how effective are the various interventions to do so likely to be (according to the model)?

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Bernoulli was a Swiss mathematician famous for the kinetic theory of gasses, the Bernoulli effect in fluid flow, and early work on the statistical characterization of risk.

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Prior infection confers lifetime immunity; inoculation with *variola minor* (less fatal than *variola major*) first documented in China during the 10th century.

# Smallpox

Bangladeshi child infected with smallpox in 1973. Freedom from smallpox was declared in Bangladesh in December, 1977 when a WHO International Commission officially certified that smallpox had been eradicated from that country. The CDC declared smallpox eradicated worldwide in 1980 [Wikipedia; photo source CDC].

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Technique varied in how the target patient was prepared, what other treatments (many bogus) were combined, and how the target was exposed (scratches, deep cuts, inhalation of powdered scab, etc.).

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Cowpox is mild in humans, does not pose risk of fatality, and is not easily transmitted between humans.

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He also assumed a cohort $w(t)$ of age $t$ consisted of the never infected $x(t)$ and those with immunity $z(t)$, thus $w(t) = x(t) + z(t)$, and that the probability of those in $x(t)$ acquiring smallpox at any is always $b$ independent of $t$.

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Using $a = 0.125$ and $b = 0.125$ (estimated from observational data) he calculated that the population would be 14% larger at age 26, and that life expectancy would increase from 26.58 to 29.75 if all children were variolated at birth.

Repeating the calculation with the assumption that 2% of variolated children would die reduces the gain in life expectancy by 1 month: still a good deal for society (and the King, who wanted to increase the pool of military recruits).

Repeating the calculation again, adding the effect of secondary "artificial smallpox" infections from variolated children (recall these are likelier to be mild cases by construction) does not appreciably change these results.

# Bernoulli's Result



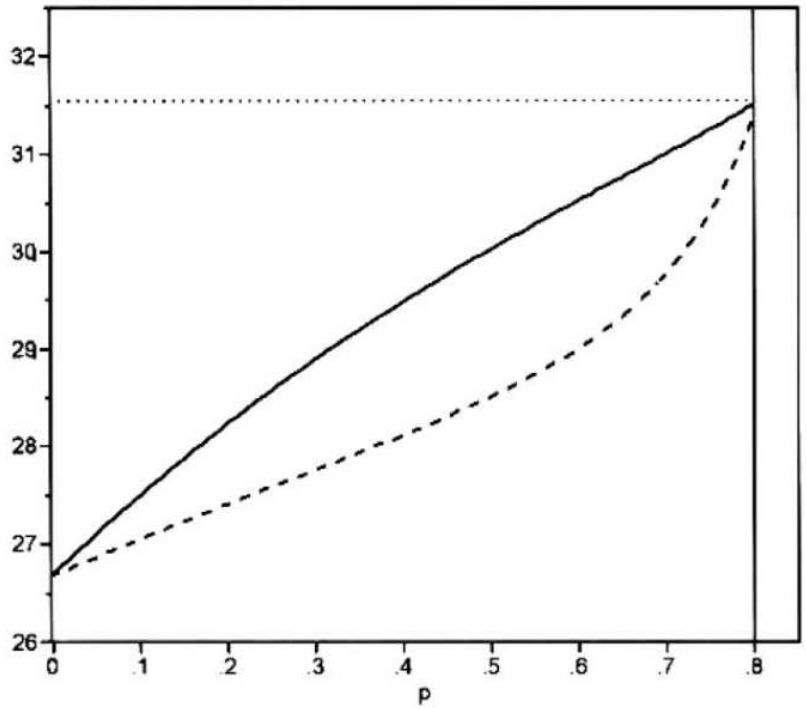


Fig. 12. Life expectancy at birth as a function of the proportion immunized $p$. The dotted line refers to those that are immunized, the broken line correponds to the non-immunized individuals and the continuous line represents the weighted average of the two values where the weight is given by $p$.

# What's Next?

Neither Bernoulli nor Snow needed a computer: their models was simple enough that it could be solved by hand (Bernoulli) or by visualization and simple statistics (Snow) while still providing insight in the underlying disease process.

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In the next lecture, we're going to look at a recent paper that is similar but much more data driven.