Homework 14 **Name:** ______________________________

Biostatistics (STAT:3510; Bognar)

Print this pdf file (do not use notebook paper), show your work in the provided space, use a <u>*scanning app*</u> *to scan pages* (<u>*in order*</u>) *into a* <u>*single pdf file*</u>*, submit in Gradescope. Be sure to get* <u>*entire page*</u> *in each shot — lay each page* <u>*flat*</u> *when scanning. You can use an iPad/tablet too. The Gradescope app works well for submitting too. Make sure the pages upload* <u>*in order*</u>*.*

1. At a large hospital, the salaries (y, in thousands of dollars) and years of experience (x) of six randomly chosen female nurses are

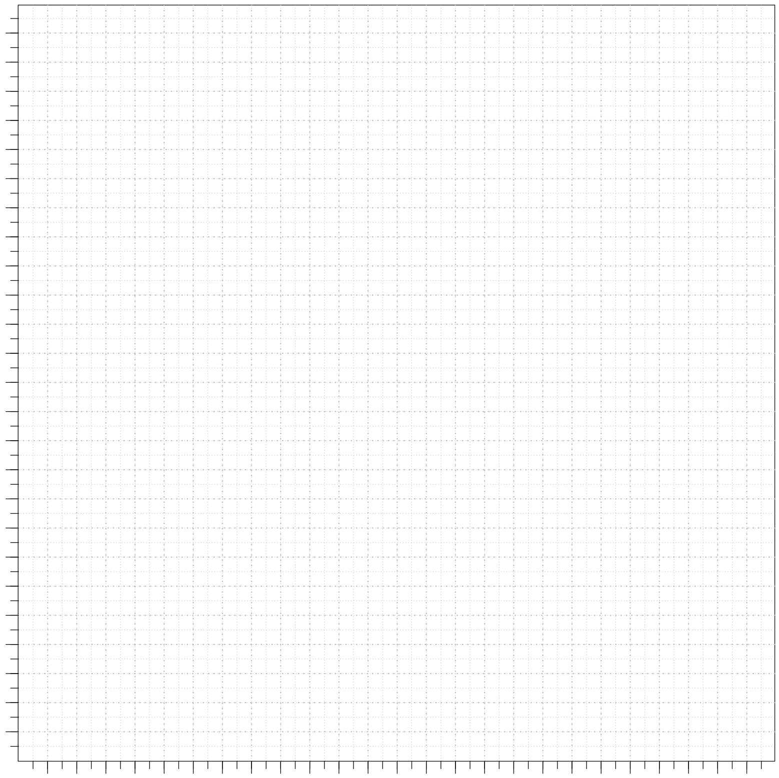
$x =$ experience:	6	7	9	10	13	15
$y =$ salary:	40	41	43	45	46	49

The R output is shown in at the end of this document.

(a) By hand, compute Pearsons sample correlation coefficient r. *Make sure your work matches* $\bar{x}$, $\bar{y}$, s_x, s_y, $Cov(x, y)$, *and* r *on the R output below.*

(b) By hand, determine least squares regression line. *Find* $\hat{\beta}_0$ *and* $\hat{\beta}_1$ *on the R output.*

(c) Carefully make a scatter-plot of the dataset and draw the regression line (place the explanatory variable x on the horizontal axis, and the response variable y on the vertical axis).



(d) On average, each extra year of experience yields how much extra pay?

(e) What is the approximate average starting pay?

(f) Approximate the mean salary for female nurses with 12 years of experience, i.e. approximate $\mu_{y|x=12}$.

(g) Approximate the mean salary for female nurses with 6 years of experience, i.e. approximate $\mu_{y|x=6}$.

(h) By hand, find a 95% confidence interval for the population mean salary of female nurses with 6 years of experience, i.e. find a 95% CI for $\mu_{y|x=6}$. Interpret the CI. *Hint: According to R,* $\widehat{se}(\hat{y}) = 0.448$. *See if you can find* $\hat{y}$, $\widehat{se}(\hat{y})$, *and the CI on the R output.*

(i) Is there a significant linear relationship between years of experience and salary? *Hint: According to R,* $\widehat{se}(\hat{\beta}_1) = 0.0878$. *You must state* H_0 *and* H_a *(use* $\alpha = 0.05$*), find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion. See if you can find* $\hat{\beta}_1$, $\widehat{se}(\hat{\beta}_1)$*, and the test statistic* t^* *on the R output.*

(j) Approximate the $p-$value for the test in 1i using the $t-$table. Based upon your $p-$value, is there a significant linear relationship between years of experience and salary? Why?

(k) Use the $t-$Probability Applet at

`http://www.stat.uiowa.edu/~mbognar/applets/t.html`

to precisely determine the $p-$value for the test in 1i. *See if you can find* $p-$*value for this test on the R output.*

(l) Find a 95% confidence interval for β_1. Based upon your CI, is there a significant linear relationship between years of experience and salary? Why? *Hint: According to R,* $\widehat{se}(\hat{\beta}_1) = 0.0878$. *See if you can find* $\hat{\beta}_1$ *and* $\widehat{se}(\hat{\beta}_1)$ *on the R output.*

(m) Find a 95% confidence interval for the (population) mean starting salary, i.e. find a 95% CI for $\beta_0 = \mu_{y|x=0}$. *Hint: According to R,* $\widehat{se}(\hat{\beta}_0) = 0.9208$. *See if you can find* $\hat{\beta}_0$ *and* $\widehat{se}(\beta_0)$ *on the R output.*

(n) In reference to question 1m, is the population mean starting salary significantly different than 40 (i.e. $40,000)? Why?

(o) By hand, find the coefficient of determination, R^2. Interpret. *See if you can find R^2 on the R output.*

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======================================
Analysis of the salary dataset using R
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> x <- c(6,7,9,10,13,15)
> y <- c(40,41,43,45,46,49)

> mean(x)
[1] 10
> sd(x)
[1] 3.464102
> mean(y)
[1] 44
> sd(y)
[1] 3.34664
> cov(x,y)
[1] 11.4
> cor(x,y)
[1] 0.9833434

> salary.results <- lm(y~x)
> summary(salary.results)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  34.5000     0.9208   37.47 3.03e-06
x             0.9500     0.0878   10.82 0.000414

Residual standard error: 0.6801 on 4 degrees of freedom
Multiple R-squared:  0.967, Adjusted R-squared:  0.9587
F-statistic: 117.1 on 1 and 4 DF,  p-value: 0.0004139

> anova(salary.results)
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value    Pr(>F)
x          1  54.15  54.150  117.08 0.0004139
Residuals  4   1.85   0.463

> predict(salary.results, list(x=c(6)), interval="confidence", se.fit=TRUE)
$fit
   fit      lwr      upr
1 40.2 38.95704 41.44296
$se.fit
[1] 0.448
```

2. A new speech processor for existing cochlear implants has been developed. The population regression equation is

$$\mu_{y|x_1,x_2} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

where y denotes the speech recognition score (i.e. y is simply the percentage of spoken words correctly understood), x_1 denotes the subjects age, and x_2 equals 0 if the subject is using the old processor, and 1 if using the new processor. A clinical trial involving 23 subjects compared the new processor to the old processor. The following multiple regression model was used:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2$$

The analysis yielded the following results in R:

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
Intercept    43.1376    12.6781   3.403   0.0028
age          -0.1034     0.8763  -0.118   0.9072
processor    22.3329     8.1293   2.747   0.0124
```

Note that $\hat{\beta}_0 = 43.1376$, $\widehat{se}(\hat{\beta}_0) = 12.6781$, $\hat{\beta}_1 = -0.1034$, $\widehat{se}(\hat{\beta}_1) = 0.8763$, $\hat{\beta}_2 = 22.3329$, and $\widehat{se}(\hat{\beta}_2) = 8.1293$.

(a) Is age significantly associated with the mean speech recognition score? To answer this, test $H_0 : \beta_1 = 0$ vs $H_a : \beta_1 \neq 0$ at the $\alpha = 0.05$ significance level (find the test statistic and critical value, plot the rejection region, and state your decision and final conclusion).

(b) Find the $p-$value for the test in part (a). Based upon the $p-$value, is age significantly associated with the mean speech recognition score? Why?

(c) Find a 95% confidence interval for β_1. Based upon the CI, is age significantly associated with mean speech recognition score? Why?

(d) Find a 95% confidence interval for β_2. Is there a significant difference in mean speech recognition score between the old and new processors? Why?

(e) If we were to test $H_0 : \beta_2 = 0$ vs $H_a : \beta_2 \neq 0$ at the $\alpha = 0.05$ significance level, would the $p-$value be less than 0.05 or more than 0.05? Why? Base your answer on the CI in part (d).

(f) Find the $p-$value for the test in part (e). Is there a significant difference in mean speech recognition score between the old and new processors? Why?

(g) Approximate the mean speech recognition score for 33 year olds with the new processor, i.e. approximate $\mu_{y|x_1=33,x_2=1}$.

(h) In part (g), R determined that $\widehat{se}(\hat{y}) = 5.77$. Compute a 95% confidence interval for $\mu_{y|x_1=33,x_2=1}$.

(i) Is the mean speech recognition score for 33 year olds with the new processor significantly higher than 50? Why?