

Limits of Computation (CS:4340:0001 or 22C:131:001)
Homework 6

The homework is due in class on Thursday, May 7. If you can't make it to class, drop it in my mailbox in the MacLean Hall mailroom.

1. Describe a polynomial time algorithm that given a quantified boolean formula (QBF) ψ outputs an 'equivalent' QBF ψ' in prenex normal form. Illustrate your algorithm using an example or two. (The algorithm is briefly given in Section 4.2, in the penultimate paragraph on page 83.) Formally argue the equivalence for at least one of the 3 cases (and, or, and not). (3 points)
2. Describe a polynomial time algorithm that given a QBF ψ in prenex normal form outputs an equivalent QBF ψ' also in prenex normal form such that (a) the quantifiers in ψ' alternate between \exists and \forall as we go from left to right, and (b) the leftmost quantifier and the rightmost quantifier are \exists . Illustrate using an example. (1 point)
3. Let L be a language that is not the empty set or $\{0, 1\}^*$. Argue that for any language L' in the class \mathbf{NL} , $L' \leq_P L$. (3 points)
4. Review the proof of Lemma 6.11, which asserts that $\text{CKT-SAT} \leq_P 3\text{SAT}$. Show how the reduction algorithm works on the following circuit. (3 points) Abbreviate how the gates are translated into CNF, to avoid writing a huge answer.

