Limits of Computation (CS:4340:0001 or 22C:131:001)
Homework 4

The homework is due in class on Tuesday, April 7th. If you can't make it to class, drop it in my mailbox in the MacLean Hall mailroom.

Each of the following four questions is worth 2.5 points.

1. We have defined a relation $\leq_p$ among langauges. We noted that it is *reflexive* (i.e., $L \leq_p L$ for all languages L) and *transitive* (i.e., if $L \leq_p L'$ and $L' \leq_p L''$ then $L \leq_p L''$). Show that it is not *symmetric*, namely, $L \leq_p L'$ need not imply $L' \leq_p L$. (This is Exercise 2.9 in the text.)

2. We saw that coNP could be defined in two ways:

   (a) A language L belongs to coNP if its complement $\overline{L}$ belongs to NP.

   (b) A language L belongs to coNP if there exists a polynomial $p : \mathbb{N} \to \mathbb{N}$ and a polynomial time TM M such that for every $x \in \{0,1\}^*$,

   $$x \in L \Leftrightarrow \forall u \in \{0,1\}^* \text{ such that } |u| \leq p(|x|), M(x,u) = 1.$$

   Show that the two definitions are equivalent, that is, they define the same class of languages. (This is basically Exercise 2.24.)

3. Prove that if P = NP, then NP = coNP. (This is Exercise 2.25.)

4. Show that NP = coNP if and only if `3SAT` and `TAUTOLOGY` are polynomial-time reducible to each other. (If you prefer, you can replace `TAUTOLOGY` by $\overline{\texttt{3SAT}}$ in the problem statement.) (This is Exercise 2.26.)