Limits of Computation (CS:4340:0001 or 22C:131:001)
Homework 3

The homework is due in class on Tuesday, March 10th. If you can't make it to class, drop it in my mailbox in the MacLean Hall mailroom.

Each question is worth 2.5 points.

1. Define a TM M to be *oblivious* if its head movements do not depend on the input but only on the input length. That is, M is oblivious if for every input $x \in \{0,1\}^*$ and integer $i \geq 0$, the location of each of M's heads at the i'th step of execution on input x is only a function of $|x|$ and i. Show that for every time-constructible function T, if $L \in DTIME(T(n))$ then there is an oblivious TM that decides L in $O(T(n)^2)$ time. Furthermore, show that there is such a TM that uses only two tapes: one input tape and one work/output tape. **Hint:** Modify the method in Claim 1.6. (This problem is Exercise 1.5 in the text.)

2. Consider the language $\texttt{HALT} = \{\langle \beta, x \rangle | M_\beta \text{ halts on } x\}$. Show that `HALT` is NP-hard. Is it NP-complete? Explain. (This is Exercise 2.8 in the text.)

3. Suppose $L_1, L_2 \in NP$. Then is $L_1 \cup L_2 \in NP$? What about $L_1 \cap L_2$? Explain. (This is Exercise 2.10 in the text.)

4. Study Section 2.4 (The Web of Reductions), where the languages `INDSET`, `0/1 IPORG`, and `dHAMPATH` are shown to be NP-complete. Then explain any one of these three reductions in your own words.