$22\mathbf{C} : 031$ ($CS : 3310 : 0001$) **Algorithms**
Homework 2

- We have examined a few problems for which we were able to find a greedy algorithm that works. In the process, we also showed that several other rather natural looking greedy algorithms do not always work. However, some nice things can be said about some of these algorithms.

  Let us revisit one of these greedy algorithms for the interval scheduling problem (Section 4.1): Pick the shortest (that is, the minimum length) interval, remove it and all incompatible intervals, and repeat this process till no more intervals remain.

  For this greedy algorithm, argue that the number of intervals picked is at least as large as half the size of the optimal solution. (15 points)

  Hint: Let us recall the analysis of the greedy algorithm that picks the interval with the earliest finish time, rephrasing it slightly so that it suggests a direction for the argument in the present question. Suppose that the optimal solution consists of the intervals $o_1, o_2, \ldots, o_m$. The analysis works by claiming that after the first iteration, when the greedy algorithm has picked its first interval and removed incompatible intervals, *at most one* of the intervals in the optimal solution has been discarded, so *at least* $m-1$ of the intervals in the optimal solution are still in the set of intervals that have not been discarded. After the first two iterations, *at most two* of the intervals in the optimal solution have been discarded, so *at least* $m-2$ of the intervals in the optimal solution are still in the set of intervals that have not been discarded.

- A k-coloring of an undirected graph is an assignment of a number (called a color) from the set $\{1, 2, \ldots, k\}$ to each vertex of the graph, with the property that for any edge, the two incident vertices receive different colors. We talked about graph coloring in the context of the interval partitioning problem from Section 4.1. Recall that the *degree* of a vertex u in a graph is the number of edges incident to u. Describe an algorithm that $(\Delta + 1)$-colors a given undirected graph $G = (V, E)$, where Δ is the maximum degree of a vertex in the graph. (We are looking for an efficient algorithm, in particular, it should run in time that is polynomial in the number of vertices plus edges of the graph. Try some examples to see how you would do the coloring.) (15 points)

- Exercise 7 of Chapter 4. (15 points)

- Exercise 12 of Chapter 4. (15 points)

The homework is due Thursday, February 17, in class; if you can't make it to class on that day, just make sure you get it to me by that time.