Limits of Computation (CS:4340:0001 or 22C:131:001)
Homework 3

The homework is due by class time on Tuesday, October 13th. For this homework, I will create a dropbox in ICON called `Homework3`. Please submit an electronic copy in the dropbox. It is fine to submit a scanned copy of a handwritten solution.

Each question is worth 2.5 points.

1. An undirected graph $G = (V, E)$ is said to be *bipartite* if its vertex set V can be partitioned into two sets A and B such that every edge in E is from a vertex in A to a vertex in B (there is no edge between two members of A or two members of B). Describe a polynomial-time algorithm to determine if an input graph G is bipartite.

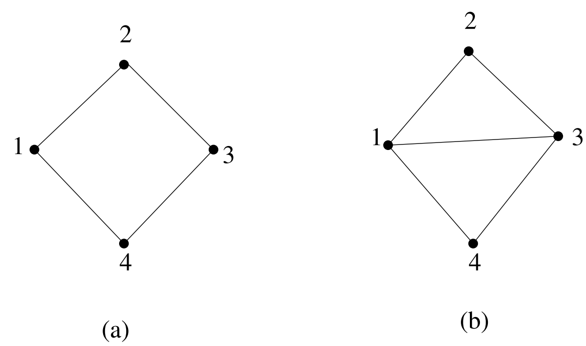

Figure 1: (a) A bipartite graph, as can be seen by setting $A = \{1, 3\}$ and $B = \{2, 4\}$; (b) A non-bipartite graph

For this problem, feel free to look up the algorithm, and then describe it in your own words. (For those whole like to be challenged, however, many of you have the algorithmic background to figure this out on your own.)

2. Consider the language $\texttt{HALT} = \{\langle \beta, x\rangle | M_\beta \text{ halts on } x\}$. Show that `HALT` is NP-hard. Is it NP-complete? Explain. (This is Exercise 2.8 in the text.)

3. Suppose $L_1, L_2 \in NP$. Then is $L_1 \cup L_2 \in NP$? What about $L_1 \cap L_2$? Explain. (This is Exercise 2.10 in the text.)

4. In the `MAX-CUT` problem, we are given an undirected graph $G = (V, E)$ and an integer K and have to decide if there is a subset $S \subseteq V$ of the vertices such that there are at least K edges that have one endpoint in S and the other in $V - S$.

   For illustration, take the graph in Figure 1 (b). If $S = \{1, 2\}$, there are three edges with one endpoint in S and the other in $V - S$. If $S = \{1, 3\}$, there are four edges with one endpoint in S and the other in $V - S$.

Prove that the `MAX-CUT` is NP-complete. You can learn a lot about doing reductions if you attempt to solve this problem on your own. However, for this problem, you are free to lookup the solution and then describe it in your own words.