HW 9 [PG]. Page 254: Take a look at problems 1 to 8 and try to answer these questions by reading the chapter carefully. You do not need to turn in these problems.
Turn in the following exercises: Page 254-256: Chapter 10, Exercises 9, 11, 12, 14.

9. a. The null hypothesis is $H_0 : \mu = 74.4\ mmHg$. b. The alternative hypothesis is $H_A : \mu \neq 74.4\ mmHg$. c. The test statistic is

$$z = \frac{84 - 74.4}{9.1/\sqrt{10}} = 3.34.$$

The p-value $p < 0.002$. d. Since $p < 0.002$, we reject the null hypothesis. In fact, the diastolic blood pressure for the female diabetics tend to be higher. e. Since $p < 0.01$, the conclusion would be the same.

11. a. We use the t-distribution with $df = 58 - 1 = 57$. A two-sided confidence interval is:

$$(25.0 - 2.000 \times \frac{2.7}{\sqrt{58}}, 25.0 + 2.000 \times \frac{2.7}{\sqrt{58}}) = (24.3, 25.7).$$

b.

$$H_0 : \mu = 24.0, \quad H_A : \mu \neq 24.0.$$

The test statistic is

$$\begin{aligned} t &= \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \\ &= \frac{25.0 - 24.0}{2.7/\sqrt{58}} \\ &= 2.82. \end{aligned}$$

The p-value: $0.001 < p < 0.01$. Therefore, we reject H_0.

c. The mean baseline body mass index for the population of men who later developed diabetes mellitus is higher than $24.0 kg/m^2$.

d. Since the value 24.0 is not contained in the 95% confidence interval for μ, we would expect that the null hypothesis would be rejected.

12. a.

$$H_0 : \mu = 136\ mmHg, \quad H_A : \mu \neq 136\ mmHg.$$

The test statistic is

$$\begin{aligned} t &= \frac{\bar{x}_s - \mu_{0s}}{s_s/\sqrt{n}} \\ &= \frac{143 - 136}{24.4/\sqrt{86}} \\ &= 2.66. \end{aligned}$$

For a t-distribution with 85 df, $0.001 < p < 0.01$. We reject H_0 at the 0.10 level of significance.

b.

$$H_0 : \mu = 84\ mmHg, \quad H_A : \mu \neq 84\ mmHg.$$

The test statistic is

$$\begin{aligned} t &= \frac{\bar{x}_d - \mu_{0s}}{s_d/\sqrt{n}} \\ &= \frac{87 - 84}{16.0/\sqrt{86}} \\ &= 1.74. \end{aligned}$$

For a t-distribution with 85 df, $0.05 < p < 0.10$. We reject H_0 at the 0.10 level of significance.

c. The workers who have experienced a major coronary event have a higher level of mean systolic blood pressure and a higher level of diastolic blood pressure than workers who do not.

14. a. The typy I error rate $= \alpha = 0.05$. b. First, the critical value $\bar{x}$ satisfies:

$$\frac{\bar{x} - 244}{41/\sqrt{n}} = -1.645.$$

Therefore,

$$\bar{x} = 230.5.$$

That is, the null hypothesis will be rejected if the sample has a mean no greater than 230.5 mg/100ml. Under the alternative hypothesis value $\mu = 219$, we have

$$z = \frac{230.5 - 219}{41/\sqrt{25}} = 1.40.$$

The area to the right of 1.40 under the standard normal curve is 0.081. Therefore, the type II error rate $\beta = 0.081$.

c. The power

$$\text{power} = 1 - \beta = 1 - 0.081 = 0.919.$$

d. The power can be increased by increasing the sample size, by increasing the type I error rate α, or by considering a hypothesized mean that is larger than 244 that is further away from 219.

e. For $\alpha = 0.05$ and $\beta = 0.05$, the required sample size

$$n = \left[\frac{(z_\alpha + z_\beta)\sigma}{\mu_1 - \mu_0}\right]^2 = \left[\frac{(1.645 + 1.645)41}{244 - 219}\right]^2 = 29.1.$$

Thus sample size of 30 would be needed.

f. If the type I error rate $\beta = 0.10$, then

$$n = \left[\frac{(z_\alpha + z_\beta)\sigma}{\mu_1 - \mu_0}\right]^2 = \left[\frac{(1.645 + 1.280)41}{244 - 219}\right]^2 = 23.0.$$

Thus sample size of 23 would be needed.