

HW 7 [PG] Pages 227-229 Chapter 9, Exercises: 5, 7, 8, 9, 10

5. a. A two-sided 95% confidence interval μ_s is

$$(130 - 1.96 \times (11.8/\sqrt{10}), 130 + 1.96 \times (11.8/\sqrt{10})) = (122.7, 137.7).$$

b. Approximately 95% of the intervals constructed this way will cover μ .

c. A two-sided 90% confidence interval for μ_d is

$$(84 - 1.645 \times (11.8/\sqrt{10}), 84 + 1.645 \times (11.8/\sqrt{10})) = (79.3, 88.7).$$

d. A two-sided 99% confidence interval for μ_d is

$$(84 - 2.58 \times (11.8/\sqrt{10}), 84 + 2.58 \times (11.8/\sqrt{10})) = (76.6, 91.4).$$

e. The 99% confidence interval is wider than the 90% interval. The shorter the confidence interval, the less confident we are that the interval covers μ_d .

7. a. For the t distribution with $df = 21$, 1% lies to the left of $t = -2.518$.

b. 10% of the area lies to right of $t = 1.332$.

c. 5% lies to the left of $t = -1.721$, and another 0.5% lies to the right of $t = 2.831$. Therefore, 94.5% of the area lies between these two values.

d. The value $t = -2.080$ cuts off the lower 2.5% of the distribution.

8. a. Since the population standard deviation (σ) is unknown, we use the t -distribution with $df = 11$. A two-sided 95% confidence interval for μ_1 is

$$(4.49 - 2.201 \times (0.82/\sqrt{12}), 4.49 + 2.201 \times (0.82/\sqrt{12})) = (3.96, 5.02).$$

b. A two-sided 95% confidence interval for μ_1 is

$$(4.49 - 1.796 \times (0.82/\sqrt{12}), 4.49 + 1.796 \times (0.82/\sqrt{12})) = (4.06, 4.92).$$

The 90% confidence interval is shorter than the 95% interval.

c. A two-sided 95% confidence interval for μ_2 is

$$(3.71 - 2.201 \times (0.62/\sqrt{12}), 3.71 + 2.201 \times (0.62/\sqrt{12})) = (3.32, 4.10).$$

d. It is assumed that the underlying distributions of FVC and FEV₁ are approximately normal.

9. a. We use the t distribution with $df = 13$. A two-sided 95% confidence interval for μ is

$$(29.6 - 2.160 \times (3.6/\sqrt{14}), 29.6 + 2.160 \times (3.6/\sqrt{14})) = (27.5, 31.7).$$

b. The length of this interval is $31.7 - 27.5 = 4.2$ weeks.

c. We are interested in the sample size necessary to produce the confidence interval

$$(29.6 - 1.5, 29.6 + 1.5).$$

We know that the 95% confidence interval has the form

$$(29.6 - 1.96 \times (3.6/\sqrt{n}), 29.6 + 1.96 \times (3.6/\sqrt{n})).$$

So we have

$$1.96 \times (3.6/\sqrt{n}) = 1.5,$$

or

$$n = \left[\frac{1.96 \times 3.6}{1.5} \right]^2 = 22.1.$$

A sample of size 23 is needed.

d. As in part c, we have

$$1.96 \times (3.6/\sqrt{n}) = 1.0.$$

So

$$n = \left[\frac{1.96 \times 3.6}{1.0} \right]^2 = 49.8.$$

A sample of size 50 is needed.

10. a. We have

$$\bar{x} = 112.8\%, \quad s = 14.4\%.$$

A 95% confidence interval for the mean percentage of ideal body weight is

$$(112.8 - 2.110 \times (14.4/\sqrt{18}), 112.8 + 2.110 \times (14.4/\sqrt{18})) = (105.6, 120.6).$$

b. The confidence interval does not contain 100%. As a result, we conclude that the mean percentage of ideal body weight for the population of insulin dependent diabetics is different from 100%; the true percentage is higher.