HW 6: [PG] Pages: 210-212 Chapter 8, Exercises: 9, 11, 13

9. a. The distribution of the means of samples of size 10 has mean $\mu = 0$ and standard error $\sigma/\sqrt{10} = 1/\sqrt{10} = 0.32$, and is normally distributed.

b. The proportion of means that are greater than 0.60 is

$$\begin{aligned} P(\bar{X} > 0.60) &= P\left(\frac{\bar{X}-0}{0.32} > \frac{0.60-0}{0.32}\right) \\ &= P(Z > 1.87) \\ &= 0.031 = 3.1\%. \end{aligned}$$

c. The proportion of means that are less than -0.76 is

$$\begin{aligned} P(\bar{X} < -0.76) &= P\left(\frac{\bar{X}-0}{0.32} < \frac{0.76-0}{0.32}\right) \\ &= P(Z < -2.34) \\ &= 0.010 = 1.0\%. \end{aligned}$$

d. The value $z = 0.84$ cuts off the upper 20% of the standard normal distribution. Therefore, $\bar{x} = 0.84 \times 0.32 + 0 = 0.27$ cuts off the upper 20% of the distribution of sample means.

e. The value $z = -1.28$ cuts off the lower 10% of the standard normal distribution. Therefore, $\bar{x} = -1.28 \times 0.32 + 0 = -0.41$ cuts off the lower 10% of the distribution of sample means.

11. a. The probability that the newborn's birth weight is less than 2500 grams is

$$\begin{aligned} P(X < 2500) &= P\left(\frac{X-3500}{430} < \frac{3500-2500}{430}\right) \\ &= P(Z < -2.34) \\ &= 0.010 = 1.0\%. \end{aligned}$$

b. The value $z = -1.645$ cuts off the lower 5% of the standard normal distribution. Therefore, $x = -1.645 \times 430 + 3500 = 2793$ cuts off the lower 5% of the distribution of birth weights.

c. The distribution of means of samples of size 5 had mean $\mu = 3500$ and standard error $\sigma/\sqrt{n} = 430/\sqrt{5} = 192$ grams, and is approximately normally distributed.

d. The value $\bar{x} = -1.645 \times 192 + 3500 = 3184$ cuts off the lower 5% of te distribution of samples of size 5.

e. The probability that the sample mean is less than 2500 grams is

$$\begin{aligned} P(\bar{X} < 2500) &= P\left(\frac{X-3500}{192} < \frac{3500-2500}{192}\right) \\ &= P(Z < -5.12) \\ &= 0.0000. \end{aligned}$$

f. The number of newborns with birth weight less than 2500 grams follows a binomial distribution with $n = 5$ and $p = 0.01$. Thus the probability of only one of the 5 newborns has a birth weight less than 2500 grams is

$$P(X = 1) = \binom{5}{1} (0.01)^1(1 - 0.01)^{5-1} = 0.048.$$

13. a. We have

$$\begin{aligned} P(300 \le X \le 400) &= P\left(\frac{300 - 341}{79} \le \frac{\bar{X} - 341}{79} \le \frac{400 - 341}{79}\right) \\ &= P(-0.52 \le Z \le 0.75) \\ &= 1 - 0.302 - 0.227 \\ &= 0.471. \end{aligned}$$

Approximately 47.1% of the males have a serum acid level between 300 and 400 μmol/l.

b. The mean $\mu = 341$ and standard error $\sigma/\sqrt{n} = 70/\sqrt{5} = 35.3\mu$mol/l. We have

$$\begin{aligned} P(300 \le X \le 400) &= P\left(\frac{300 - 341}{35.3} \le \frac{\bar{X} - 341}{35.3} \le \frac{400 - 341}{35.3}\right) \\ &= P(-0.52 \le Z \le 0.75) \\ &= 1 - 0.123 - 0.047 \\ &= 0.830. \end{aligned}$$

Approximately 83% of the samples have a mean serum acid level between 300 and 400 μmol/l.

c. The mean $\mu = 341$ and standard error $\sigma/\sqrt{n} = 70/\sqrt{10} = 25.0\mu$mol/l. We have

$$\begin{aligned} P(300 \le X \le 400) &= P\left(\frac{300 - 341}{25.0} \le \frac{\bar{X} - 341}{25.0} \le \frac{400 - 341}{25.0}\right) \\ &= P(-1.64 \le Z \le 2.36) \\ &= 1 - 0.051 - 0.009 \\ &= 0.940. \end{aligned}$$

Approximately 94% of the samples of size 10 have a mean serum acid level between 300 and 400 μmol/l.

d. For the standard normal distribution, the interval $(-1.96, 1.96)$ contains 95% of the observations. There, the interval

$$(-1.96 \times 25.0 + 341, 1.96 \times 25.0 + 341) = (292, 390)$$

contains 95% of the means of samples of size 10. This symmetric interval is shorter than an assymmetric one.