HW 5. [PG]

Chapter 7, Pages 192-194: 11, 16, 17, 19, 20

Chapter 8, Pages 210-211: 4, 6, 8

Ch7 11. a. $10! = 3,628,800$ different ways.

b.

$$\left(\begin{array}{c} 10 \\ 4 \end{array} \right) = \frac{10!}{4!}6! = 210.$$

c. The probability that exactly three of the ten individuals are left-handed is

$$P(3\text{left-handed}) = \left(\begin{array}{c} 10 \\ 3 \end{array} \right) 0.098^3 \times 0.902^7 = 0.055.$$

d. The probability that at least 6 three of the ten individuals are left-handed is

$$\begin{aligned} P(\text{at least}6) &= \sum_{x=6}^{1} 0 \left(\begin{array}{c} 10 \\ x \end{array} \right) (0.098^x)(0.902^{10-x}) \\ &= \left(\begin{array}{c} 10 \\ 6 \end{array} \right) (0.098^6)(0.902^4) + \left(\begin{array}{c} 10 \\ 7 \end{array} \right) (0.098^7)(0.902^3) \\ &\quad + \left(\begin{array}{c} 10 \\ 8 \end{array} \right) (0.098^8)(0.902^2) + \left(\begin{array}{c} 10 \\ 9 \end{array} \right) (0.098^9)(0.902^1) \\ &\quad + \left(\begin{array}{c} 10 \\ 10 \end{array} \right) (0.098^1 0)(0.902^0) \\ &= 0.0001. \end{aligned}$$

e. The probability that at most two are left-handed is

$$\begin{aligned} P(\text{at most}2) &= P(none) + P(one) + P(two) \\ &= \left(\begin{array}{c} 10 \\ 0 \end{array} \right) (0.098^0)(0.902^1 0) + \left(\begin{array}{c} 10 \\ 1 \end{array} \right) (0.098^1)(0.902^9) \\ &\quad + \left(\begin{array}{c} 10 \\ 2 \end{array} \right) (0.098^2)(0.902^8) \\ &= 0.933. \end{aligned}$$

Ch 7, 16. a. Use the Poisson approximation to the binomial distribution. The mean number of infants who would die in the first year is $\lambda = np = 2000 \times 0.0085 = 17$.

b. The probability that at most 5 infants die in the first year is

$$\begin{aligned} P(\text{at most}5) &= P(0) + P(1) + \cdots + P(5) \\ &= \sum_{x=0}^{5} e^{-17} \frac{17^x}{x!} \\ &= 0.00067. \end{aligned}$$

c. The probability that between 15 and 20 infants die in the first year of life is

$$\begin{aligned} P(\text{between15and20}) &= \sum_{x=15}^{2} 0e^{-17}\frac{17^x}{x!} \\ &= 0.525. \end{aligned}$$

Ch 7, 17. a. 0.005.

b. 0.911

c. 1-0.045-0.001=0.945.

d. The value $z = 1.04$ cuts off upper 15% of the standard normal distribution.

e. The value $z = -0.84$ cuts off the lower 20% of the standard normal distribution.

noindent Ch 7, 19. a.

$$\begin{aligned} P(X < 130) &= P\left(\frac{X - 172.2}{29.8} < \frac{130 - 172.2}{29.8}\right) \\ &= P(X < -1.42) \\ &= 0.078. \end{aligned}$$

b.

$$\begin{aligned} P(X > 210) &= P\left(\frac{X - 172.2}{29.8} > \frac{210 - 172.2}{29.8}\right) \\ &= P(X > 1.27) \\ &= 0.102. \end{aligned}$$

c.

$$\begin{aligned} \text{at least one} < 130 \text{ or } < 210) &= 1 - P(\text{none} < 130 \text{ or } > 210) \\ &= 1 - P(\text{all between130 and 210}) \\ &= 1 - [P(130 \le X \le 210)]^5 \\ &= 1 - [1 - 0.078 - 0.102]^5 \\ &= 1 - 0.820^5 \\ &= 0.629. \end{aligned}$$

Ch7, 20. a. The probability of correctly predicting coronary heart disease for a man who will develop it is

$$\begin{aligned} P(X_d > 260) &= P\left(\frac{X - 244}{51} > \frac{260 - 244}{51}\right) \\ &= P(Z > 0.31) \\ &= 0.378. \end{aligned}$$

b. The probability of incorrectly predicting heart disease for a man who will not develop it, or the probability of false positive, is

$$\begin{aligned} P(X_{nd} > 260) &= P\left(\frac{X-219}{41} > \frac{260-219}{41}\right) \\ &= P(Z > 1.00) \\ &= 0.159. \end{aligned}$$

c. The probability of failing to predict heart disease for a man who will develop it, or the probability of false negative, is

$$\begin{aligned} P(X_d < 260) &= 1 - P(X_d > 260) \\ &= 1 - 0.378 \\ &= 0.622. \end{aligned}$$

d. If the cut-off point is lowered to 250 mg/100 ml, the probability of a false positive result would increase while the probability of a false negative would decrease.

e. Initial serum cholesterol is not very useful for predicting coronary heart disease in this population. Because the distributions for men who develop disease and those who do not develop have a large overlap, the probabilities of false positive and false negative outcomes are both very high.