

HW 11 [PG] : Pages 298-300: Chapter 12.

(a) Read Exercises 1 to 4, and try to answer the questions by reading this chapter carefully. You do not need to hand in these problems.

(b) Hand in: Exercises 5, 6, 8, 9.

5. a. For  $F_{8,16}$ , 10% of the area under the curve lies to the right of  $F = 2.09$   
 b. The value  $F = 3.89$  cuts off the upper 1% of the distribution.  
 c. Since 0.5% of the area under the curve lies to the right of  $F = 4.52$ , 99.5% lies to the left of this value.
6. a. For  $F_{3,30}$ , 0.5% of the area under the curve lies to the right of  $F = 5.24$ .  
 b. Since 5% of the area under the curve lies to the right of  $F = 2.92$ , 95% lies to the left of this value.  
 c. The value  $F = 3.59$  cuts off the upper 2.5% of the distribution.  
 d. The value  $F = 7.05$  cuts off the upper 0.1%.

8.

$$s_w^2 = 1.75,$$

$$\bar{x} = 5.58,$$

$$s_B^2 = 19.06.$$

The test statistic is

$$F = \frac{s_B^2}{s_W^2} = \frac{19.06}{1.75} = 10.89.$$

For an  $F$  distribution with  $4 - 1 = 3$  and  $1498 - 4 = 1494$  df,  $p < 0.001$ , we reject the null hypothesis.

b. We conclude that the mean LDL cholesterol level is not the same for each of the groups.

c. In order to use the one-way analysis of variance technique, the four populations must at least approximately normally distributed, and their variances must be equal.

d. It is necessary to take an additional step in this analysis. We have concluded that the mean LDL level are not the same for all four groups, but we cannot yet say which group means differ from which others. In order to do this, we can use the Bonferroni method of multiple comparisons. It would tell us that patients with intermittent claudication and those with minor asymptomatic disease have higher mean LDL levels than patients with no disease.

9. a. The average minutes of individual therapy per session is longest for the private not-for-profit centers and shortest for the for-profit centers; the average minutes of groups therapy is longest for the for-profit centers and shortest for the public centers.

b. Each 95% confidence interval takes the form  $\bar{x} \pm t_{n-1}(s/\sqrt{n})$ . The intervals are

Center	Individual Therapy	Group Therapy
FP	(44.30, 54.62)	(89.81, 121.85)
NFP	(53.49, 56.03)	(95.10, 102.26)
Public	(51.57, 54.93)	(90.00, 98.34)

For individual therapy, the three confidence intervals all overlap. Therefore, there is nothing to suggest that the population means are not the same. The same is true for group therapy.

c. To test the null hypothesis that the mean minutes of individual therapy per session are the same for each type of center, we first calculate estimates of within- and between-groups variances.

$$s_w^2 = 135.4,$$

$$\bar{x} = 53.88,$$

$$s_B^2 = 515.8.$$

The  $F$  statistic

$$F = \frac{515.8}{135.4} = 3.81.$$

For an  $F$  distribution with  $df = 3 - 1 = 2$  and  $518 - 3 = 515$ ,  $0.01 < p < 0.025$ . Thus we reject the null hypothesis at the 0.05 level of significance.

To apply the Bonferroni method of multiple comparisons, we conduct three pairwise tests at the  $0.05/3 = 0.0167$  level of significance. The test statistics are

$$t_{12} = -2.62$$

$$t_{13} = -1.79$$

$$t_{23} = 1.36.$$

All test statistics have a  $t$  distribution with 515 df. The comparison of private for-profit and not-for-profit centers results is  $p = 0.008$ ; the  $p$ -values for the other two comparisons are both greater than 0.0167. Therefore, we conclude that the mean minutes of individual therapy per session is higher for for-profit centers than for not-for-profit centers.

d. To test the null hypothesis that the mean minutes of group therapy per session are identical for each type of centers, we again calculate the  $F$  statistic.

$$s_w^2 = 947.7,$$

$$\bar{x} = 97.60,$$

$$s_B^2 = 2159.2.$$

The  $F$  statistic

$$F = \frac{2159.2}{947.7} = 2.28.$$

For an  $F$  distribution with  $3 - 1 = 2$  and  $491 - 3 = 488$ ,  $p > 0.10$ . Therefore, we are not able to reject the null hypothesis.

e. While private for-profit centers have shorter individual therapy sessions than not-for-profit centers, on average, there are no significant differences in the length of group therapy session.