HW 10 [PG].

Pages 278-280: Chapter 11, Exercises 5, 6, 8, 10

5. a. The samples are paired.

b.

$$H_0 : \mu_{corn} - \mu_{oats} = 0, \quad H_A : \mu_{corn} - \mu_{oats} \neq 0.$$

c. The sample mean of the paired differences is $\bar{d} = 0.363$, the sample standard deviation of the paired differences is $s_d = 0.406$. The t statistic is

$$t = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{0.363 - 0}{0.406/\sqrt{14}} = 3.35.$$

For a t distribution with $df = 14 - 1 = 13$, $0.001 < p < 0.01$. We reject H_0 at the 0.05 level of significance.

d. We conclude that the true difference in population mean cholesterol levels is not equal to 0. Mean LDL cholesterol level tends to be lower when individuals keep the oat bran diet.

$$z = \frac{84 - 74.4}{9.1/\sqrt{10}} = 3.34.$$

6. a.

$$H_0 : \mu_{cancer} - \mu_{control} = 0, \quad H_A : \mu_{cancer} - \mu_{control} \neq 0.$$

The data are paired. So the test statistic is

$$t = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{2.7 - 0}{15.9/\sqrt{171}} = 2.22.$$

The $df = 171 - 1 = 170$. The p-value $0.02 < p < 0.05$. We reject H_0 at the 0.05 level of significance. The mean level is higher among women with breast cancer.

b. Because the null hypothesis was rejected at 0.05 level, we would not expect the 95% confidence interval to contain the value 0.

8. a. The two samples of data are independent.

b.

$$H_0 : \mu_1 = \mu_2, \quad H_A : \mu_1 \neq \mu_2.$$

c. The pooled sample variance

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(77 - 1)(0.026)^2 + (161 - 1)(0.025)^2}{77 + 161 - 2} = 0.00064.$$

The test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_p^2(1/n_1 + 1/n_2}} = \frac{(0.098 - 0.095}{\sqrt{0.00064((1/77) + (1/161)}} = 0.86.$$

A t distribution with $df = 77+161-2 = 236$ can be approximated by the normal distribution. Therefore $p = 2 \times 0.195 = 0.390$. We do not reject H_0. This study does not provide evidence that maternal smoking has an effect on the bone mineral content of newborns.

10. a. We approximate the t distribution with 155 and 147 degrees of freedom by the t distribution with 120 df. Thus in the intervention group, a 95% confidence interval for the true mean of fate intake is

$$\bar{x}_1 \pm 1.980 \frac{s_1}{\sqrt{n_1}} = 54.8 \pm 1.980 \frac{28.1}{\sqrt{156}} = (50.3, 59.3).$$

In the control group, a 95% confidence interval for the true mean of fate intake is

$$69.5 \pm 1.980 \frac{34.7}{\sqrt{148}} = (63.9, 75.1).$$

The two confidence intervals do not overlap. (They are not plotted here, but you should plot them.) Therefore the means are not likely to be equal for the two groups.

b.

$$H_0 : \mu_1 = \mu_2, \quad H_A : \mu_1 \neq \mu_2.$$

We do not assume equal variances. The two-sample t-statistic is

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = \frac{(54.8 - 69.5}{\sqrt{28.1^2/156 + 34.7^2/148}} = -4.05.$$

The approximate degrees of freedom is

$$\begin{aligned} \nu &= \frac{[(28.1^2/156) + (34.7^2/148)]^2}{(28.1^2/156)^2/(156-1) + (34.7^2/148)^2/(148-1)]} \\ &= 282.9 \\ &\approx 282. \end{aligned}$$

A t distribution with $df = 282$ can be approximated by the normal distribution. We have $p < 0.001$ and we reject H_0 at the 0.05 level of significance. Therefore, we conclude that mean daily fat intake is not identical for the two groups of men, fat intake is lower in the intervention group.

c. A 95% confidence interval for the true difference in population means $\mu_1 - \mu_2$ is

$$\bar{x}_1 - \bar{x}_2 \pm \sqrt{(s_1^2/n_1) + (s_2^2/n_2)}$$

or

$$(54.8 - 69.5) \pm \sqrt{(28.1^2/156) + (34.7^2/148)} = (-21.8, -7.6).$$

Note that the confidence interval does not contain 0.

d. The test statistic

$$t = \frac{172.5 - 185.5}{\sqrt{(68.8^2/156) + (69.0^2/148)}} = -1.64.$$

The degrees of freedom

$$\nu = \frac{[(68.8^2/156) + (69.0^2/148)]^2}{(68.8^2/156)^2/(156-1) + (69.0^2/148)^2/(148-1)} = 301.1.$$

Thus $df = 301$. The t distribution with $df = 301$ can be approximated by the standard normal distribuion. $p = 2 \times 0.051 = 0.102$. We do not reject the null hypothesis at the 0.05 level. The data provide no evidence that mean carbohydrate intake differs for the two groups of mean.