

# Logic Design (continued)

## XOR Revisited

XOR is also called **modulo-2** addition.

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

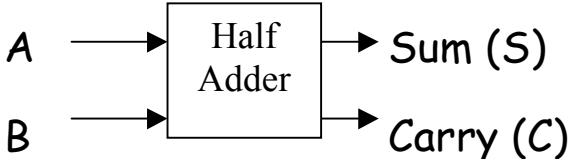
$A \oplus B = 1$  only when there are an odd number of 1's in (A,B). The same is true for  $A \oplus B \oplus C$  also.

$$\left. \begin{array}{l} 1 \oplus A = \overline{A} \\ 0 \oplus A = A \end{array} \right\}$$

Why?

# Logic Design Examples

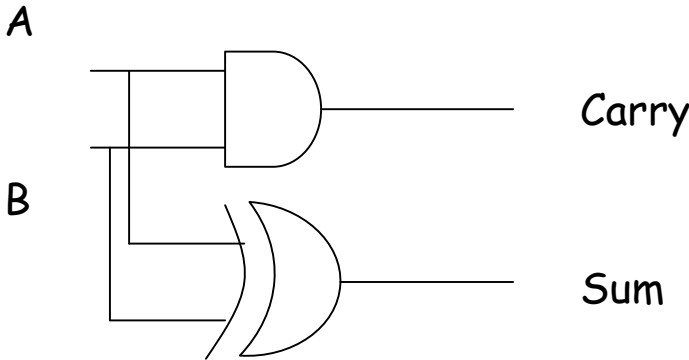
## Half Adder



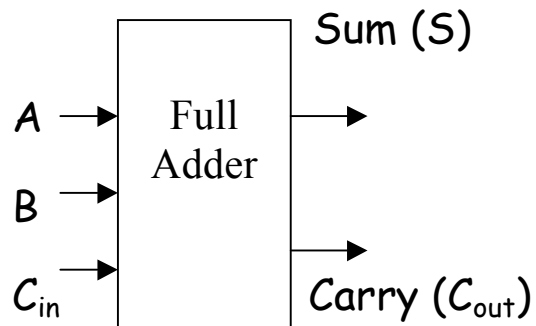
A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$S = A \oplus B$$

$$C = A.B$$



## Full Adder



A	B	C <sub>in</sub>	S	C <sub>out</sub>
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S = A \oplus B \oplus C_{in}$$

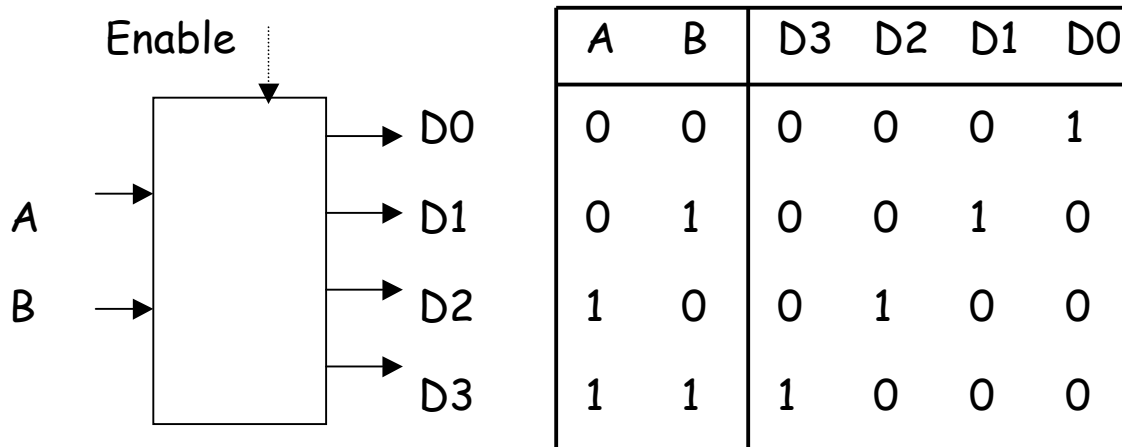
$$C_{out} = A.B + B.C_{in} + A.C_{in}$$

Design a full adder using two half-adders (and a few gates if necessary)

Can you design a 1-bit subtracter?

## Decoders

A typical decoder has  $n$  inputs and  $2^n$  outputs.



*A 2-to-4 decoder and its truth table*

$$\begin{aligned} D3 &= A.B \\ D2 &= A.\bar{B} \\ D1 &= \bar{A}.B \\ D0 &= \bar{A}.\bar{B} \end{aligned}$$

Draw the circuit of this decoder.

The decoder works per specs when (**Enable = 1**). When **Enable = 0**, all the outputs are 0.

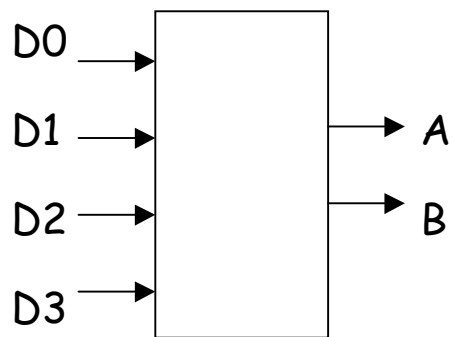
**Exercise.** Design a 3-to-8 decoder.

*Question. Where are decoders used?*

*Can you design a 2-4 decoder using 1-2 decoders?*

## Encoders

A typical encoder has  $2^n$  inputs and  $n$  outputs.



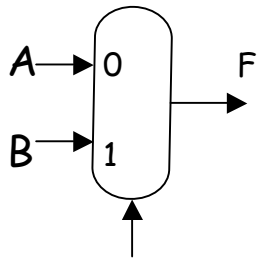
D0	D1	D2	D3	A	B
1	0	0	0	0	0
0	1	0	0	0	1
0	0	1	0	1	0
0	0	0	1	1	1

A 4-to-2 encoder and its truth table

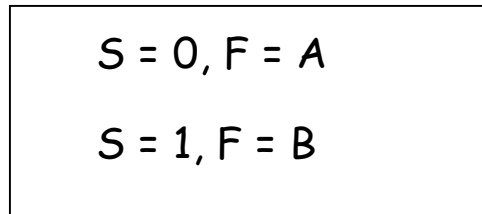
$$A = D1 + D3$$
$$B = D2 + D3$$

## Multiplexor

It is a **many-to-one switch**, also called a **selector**.



Control S



*Specifications of the mux*

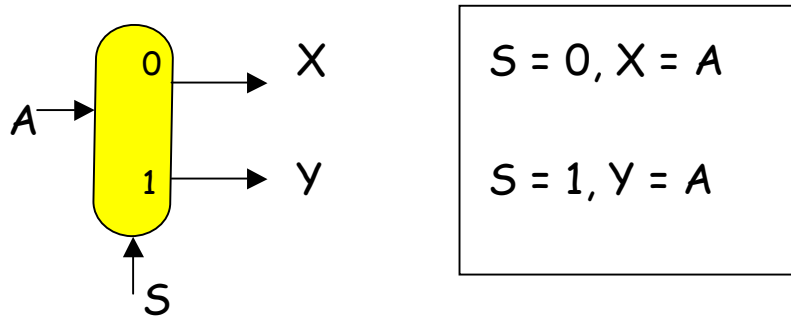
A 2-to-1 mux

$$F = \overline{S} \cdot A + S \cdot B$$

Exercise. Design a 4-to-1 multiplexor using two 2-to-1 multiplexors.

## Demultiplexors

A demux is a **one-to-many** switch.



A 1-to-2 demux, and its specification.

So,  $X = \overline{S} \cdot A$ , and  $Y = S \cdot A$

**Exercise.** Design a 1-4 demux using 1-2 demux.