Fixedpoint Study Solution

We always start with the initial approximation f_0 that is completely undefined. Then each successive approximation is generated by inserting the previous one into the functional form f(n) = **if** n=0 **then** 0 **else** n + f(n–1).

$f_0(n) = \bot$

$f_1(n)$ = **if** n=0 **then** 0 **else** n + $f_0(n–1)$ = **if** n=0 **then** 0 **else** n +⊥ = **if** n=0 **then** 0 **else** ⊥

$f_2(n)$ = **if** n=0 **then** 0 **else** n + $f_1(n–1)$ = **if** n=0 **then** 0 **else** n + (**if** (n–1)=0 **then** 0 **else** ⊥)

= **if** n=0 **then** 0 **else if** n=1 **then** 1 **else** ⊥

$f_3(n)$ = **if** n=0 **then** 0 **else** n + $f_2(n–1)$ = **if** n=0 **then** 0 **else** n + $f_2(n–1)$

= **if** n=0 **then** 0 **else** n + (**if** (n–1)=0 **then** 0 **else if** (n-1)=1 **then** 1 **else** ⊥)

= **if** n=0 **then** 0 **else if** n=1 **then** 1 **else if** n=2 **then** 3 **else** ⊥

The initial approximation (f_0) is defined nowhere. Approximation f_1 is defined only for n=0, approximation f_2 for n<2, and in general, approximation f_k is defined for n<k. Where an approximation is defined, at each stage the argument n is added into the result from the previous stage for argument n–1. So when it's defined, the result is n + (n–1) + (n–2) + … Therefore, in general

$f_k(n)$ = **if** n≤k **then** $(\sum_{i=1}^{n} I)$ **else** ⊥ = **if** n≤k **then** n*(n+1)/2 **else** ⊥ .

Since in the limit k is unbounded, lub $\{f_k\}$ = f, where f(n) =n*(n–1)/2 for all n≥0.