Sample

22C:135 — Fall 2001

Final Exam
Open book/notes

1. [30 points]

Below is a context-free grammar G (start = S) for the regular expressions over {0,1}. This grammar has terminal symbols Σ = {0,1,+,•,*,**ε**,**∅**,**(**,**)**} used in writing such regular expressions (note that the parentheses and bold font **ε** and **∅** appear as letters in the terminal alphabet here, rather than in their usual roles). The grammar provides for avoiding excessive parenthesis with the usual precedence on the regular expression operators, but no other abbreviations.

(a) provide the derivation tree for the regular expression $0•1+0^{**}$

(b) provide a convincing argument that $1^* \ 0^*$ is *not* in L(G)

S → T | S+T	T → K | T•K
K → L | K*	L → **ε** | **∅** | 0 | 1 | **(**S**)**

regular expression grammar

2. [30 points]

Consider the language L = $\{a^p b\ a^q b\ a^r \mid p,q,r \geq 0$ and either $p=r$ or $q=r\}$ over alphabet Σ = {a,b}.

(a) provide a grammar from the simplest language category (right linear, context-free, non-erasing, or unrestricted) possible, and provide a convincing argument that it generates all the strings in L and no others;

(b) prove that L is not in a simpler category than you used in (a).

3. [35 points]

Write a context-sensitive (or non-erasing) grammar for $\{a^n b^{2n} c^{3n} \mid n \geq 1\}$ and fully justify that it generates all these strings and no others.

4. [35 points]

Is it decidable, given the description encode(T) of Turing machine T, whether or not T halts for none of its inputs? Prove your answer.

5. [35 points]

For grammar G = (V, Σ, P, S) and $\alpha,\beta \in (V \cup \Sigma)^*$, is it decidable if $\alpha \Rightarrow^* \beta$,

(a) when G is context-sensitive? Prove your answer.

(b) when G is unrestricted phrase structure? Prove your answer.

6. [35 points]

For each of the languages defined below (a-g), determine the *smallest* category (i-vii) to which the language belongs (there is no implied assurance that each language falls in a distinct category).

For this problem, $\Sigma = \{0,1\}$, and encode(T) denotes a description of Turing recognizer T in the format used with the diagonal language L_d. No justification is required.

Language	Category
(a) $\{0^m1^n0^n \mid m,n \geq 1\} \cup \{10^n1^n0^m \mid m,n \geq 1\}$	(i) regular
(b) {encode(T) | T halts for one or more inputs}	(ii) deterministic context-free
(c) $\{0^p \mid p \text{ is prime}\}$	(iii) context-free
(d) $\{0^n1^n0^m \mid m,n \geq 1\} \cup \{0^m1^n0^n1 \mid m,n \geq 1\}$	(iv) context-sensitive
(e) {encode(T) | T halts for exactly one input}	(v) total Turing-recognizable
(f) $\{0^p \mid p \text{ is prime}\}^*$	(vi) (partial) Turing-recognizable (general phrase structure)
(g) {encode(T) w | T halts on w in $\text{len}(w)^{\text{len}(w)}$ steps}	(vii) all languages (i.e., other)