<u>22C:034 Discrete Structures</u>

<u>Homework VII : Solutions</u>

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Answer 1.

Suppose, the longest simple path, *P*, in a free tree *T* starts at a node *u* and ends at a node *v*.

As a free tree is a special case of undirected graph, without loss of generality, we may assume that *v* is not a leaf.

Let the above mentioned simple path is (*u, …, v', v*). As *v* is not a leaf, *v* will have a neighbor other than *v'*. Let *w* be the neighbor. But *P* being a simple path, if *w* cannot be included in *P*. Because, a free tree having no cyclic path, inclusion of *w* in *P* will make the path visit the edge (*w,v*) twice, which is unacceptable in a simple path.

So we get another simple path (*u, …, v', v, w*) which is longer than *P*. This contradicts the fact that *P* is the longest path. So our assumption that "*v* is not a leaf" is invalid. *v* must be a leaf.

Similarly we can show that *u* must be a leaf also.

[<u>Hence Proved</u>]

Answer 2.

(A)

R={(a,a),(b,b),(c,c),(d,d),(e,e),(a,c),(c,a),(a,d),(d,a),(a,e),(e,a),(b,d),(d,b),(b,e),(e,b),(c,d),(d,c),(c,e),(e,c),(d,e),(e,d)}

The above relation is reflexive and symmetric, but NOT transitive — it includes (a,c) and (c,b) but not (a,b). Hence not an Equivalence Relation.

(B)

<u>To show</u>: The above relation ***R*** in any free tree is an equivalence relation on vertices.

- **Reflexive:**
  (v,v) _ ***R***, as the path-length is 0 (even).
- **Symmetric:**
  If *u**R**v,* then there is a simple path of even length between *u* and *v*. Then because of the same path *v**R**u*.
- **Transitive:**
  Let, *u**R**v* and *v**R**w*. As there is an unique path in a tree between any two vertices, the path length of the path between *u* and *w* will be the sum or difference of the path-lengths between (*u,v*) and (*v,w*). As they are both even, the path between *u* and *w* will be of even length. Hence *u**R**w*.

Hence ***R*** is an equivalence relation on vertices of a free tree.

Answer 3.

Graph	BFS	DFS
(a)	*1,2,5,7,4,3,6*	*1,2,3,4,6,5,7*
(b)	*1,2,7,8,4,3,6,5,9*	*1,2,3,7,4,6,9,8,5*
(c)	*1,9,5,2,3,8,4,6,7*	*1,9,3,2,8,4,7,5,6*
(d)	*1,4,6,9,11,2,8,12,10,5*	*1,4,6,2,12,5,10,8,9,11*

Answer 4.
Trace of the given Graph for Dijkstra's Algorithm:

Iterations	*a*	*b*	*c*	*d*	*e*	*u*	S
0	0	4	1	INF	INF		{a}
1	-	~~4~~ 3	1	~~INF~~ 5	~~INF~~ 2	c	{a,c}
2	-	3	-	5	2	e	{a,c,e}
3	-	3	-	~~5~~ 4	-	b	{a,c,e,b}
4	-	-	-	4	-	d	{a,c,e,b,d}

Hence, The minimum distance from *a* to *d* is 4.