

Proof Rules for the Sample Language

I. Axiom of Assignment

For each assertion Q and assignment statement $X := E$

$$\vdash \{Q[X \mapsto E]\} X := E \{Q\} \text{ (ASN)}$$

II. Sequential Execution Inference Rule

For any assertions P , Q , and R and program fragments Π_1 and Π_2

$$\frac{\vdash \{P\} \Pi_1 \{Q\} \text{ and } \vdash \{Q\} \Pi_2 \{R\}}{\vdash \{P\} \Pi_1 \Pi_2 \{R\}} \text{ (SEQ)}$$

III. If-then-else Inference Rule

For any assertions P and Q , and program fragments Π_1 and Π_2

$$\frac{\vdash \{P \wedge E > 0\} \Pi_1 \{Q\} \text{ and } \vdash \{P \wedge E \leq 0\} \Pi_2 \{Q\}}{\vdash \{P\} \text{ if } E \text{ then } \Pi_1 \text{ else } \Pi_2 \text{ fi } \{Q\}} \text{ (ITE)}$$

IV. While Inference Rule

For each assertion P (called the *loop invariant*) and program fragment Π

$$\frac{\vdash \{P \wedge E > 0\} \Pi \{P\}}{\vdash \{P\} \text{ while } E \text{ do } \Pi \text{ od } \{P \wedge E \leq 0\}} \text{ (WHL)}$$

V. Strengthen Pre-condition Inference Rule

For any assertions P , Q , and R and program fragments Π

$$\frac{\vdash \{P \wedge Q\} \Pi \{R\} \text{ and } \vdash \{Q\} \Pi \{R\}}{\vdash \{P\} \Pi \{R\}} \text{ (STR)}$$

VI. Weaken Post-condition Inference Rule

$$\frac{\vdash \{Q\} \Pi \{R\} \text{ and } \vdash \{P\} \Pi \{Q\}}{\vdash \{P\} \Pi \{R\}} \text{ (WKN)}$$

Note that we do not distinguish between logically equivalent assertions. Also, since we are interested in focusing on the program proving steps, we avoid proofs about integers and take these facts as premises.