

## Homework IX

### 1. [15 points]

Consider the recursive program over all integers  $Z$

$\mathcal{P}: f(x,y) = \text{if } x > 10 \text{ then } x-1 \text{ else } f(x+2, f(x,y+1)).$

Determine the least fixed point of the functional  $\mathcal{P}$  (i.e., the recursive function defined using lazy evaluation) and justify your answer.

### 2. [20 points]

(a) Show that if  $X$  is a pointed cpo,  $C$  is a chain of  $X$ , and  $f: X \rightarrow X$  is a monotone function whose domain includes  $C$ , then  $f(C) = \{f(x) \mid x \in C\}$  is also a chain.

(b) Suppose we define a function  $\text{mp}: \text{Nat} \times \text{Nat} \rightarrow \text{Nat}$ , where  $\text{Nat}$ , and  $\text{Nat} \times \text{Nat}$  are the pointed cpos from our text, by

$$\text{mp}(\perp_{\text{Nat}}, \perp_{\text{Nat}}) = \perp_{\text{Nat}},$$

$$\text{mp}(0, \perp_{\text{Nat}}) = 0,$$

$$\text{mp}(\perp_{\text{Nat}}, 0) = 0, \text{ and}$$

$$\text{mp}(m,n) = m * n \text{ for } m,n \in \{0,1,2, \dots\}.$$

Determine whether or not  $\text{mp}$  is monotone and continuous (justify your answer).

### 3. [25 points]

Let  $\square$  be the Wren program fragment

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if A=B or A=-B
  then if A>B then C:= A else C:= B-A+1 end if
  else if A*B>0 then C:= A*B else C:= 1-A*B end if
end if

```

Assuming a store,  $\text{sto}$ , where  $\text{sto}(A)$  and  $\text{sto}(B)$  denote (positive, negative, or zero) integer values, use the denotational semantics of Wren to argue that if  $\text{execute}(\square) \text{sto} = \text{sto}'$ , then  $\text{sto}'(C) = \text{int}(c)$  with  $c > 0$ .