

Homework X

For each of the problems below, all variables involved are declared integer and assumed to contain a valid integer value. Be specific about the justifications of your steps.

1. [15 points]

Can the following be proven from the Axiom of Assignment? — why?

- (a) $\{\text{true}\} Y := X*(X+2) + 1 \{Y = (X+1)^2\}$
- (b) $\{\text{true}\} X := X+1 \{X=X+1\}$
- (c) $\{\text{false}\} X := 2 \{X=1\}$

2. [15 points]

Fill in the missing pre-conditions indicated by the question marks below to make the specifications provable with our axioms and rules of inference. The assertions that you provide should be as weak as possible, and they should not be unnecessarily complicated.

- (a) $\{?\} x := x+1; y := y + 2*x - 1 \{y=x^2\}$
- (b) $\{?\} x := x+y; y := x-y; x := x-y \{x=z \wedge y=w\}$
- (c) $\{?\} \text{ while true do skip end while } \{\text{false}\}$

3. [20 points]

Provide an axiomatic proof of partial correctness for the program fragment:

$\{F=M! \wedge 0 \leq M < N\}$
 $M := M+1; F := F*M$
 $\{F=M!\}$

4. [25 points]

Provide an axiomatic proof of partial correctness for the program fragment:

$\{\text{true}\}$
if $A=B$ **or** $A=-B$
 then if $A>B$ **then** $C := A$ **else** $C := B-A+1$ **end if**
 else if $A*B>0$ **then** $C := A*B$ **else** $C := 1-A*B$ **end if**
end if
 $\{C>0\}$