Accurately Estimating Unreported Infections using Information Theory

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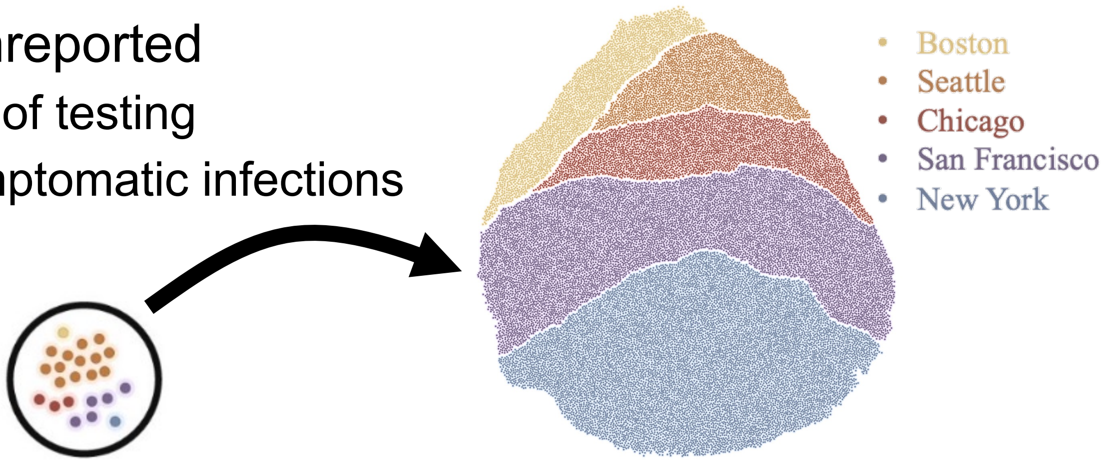

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Unreported infections

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Inability in estimating the unreported infections allowed them to drive up disease spread in the U.S. and worldwide.

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Reported rate

- Epidemiologists use reported rate (α_{reported}) to capture total infections (i.e., both reported and unreported)

- Definition:

$$\alpha_{\text{reported}} = \frac{\text{reported infections}}{\text{total infections}}$$

Reported rate estimation methods

- Serological studies: Prevalence of antibiotics
  - Accurate, but expensive and have unavoidable delays
- Influenza surveillance systems
  - Suffer from ad-hoc corrections between COVID-19 and influenza
- Epidemiological models

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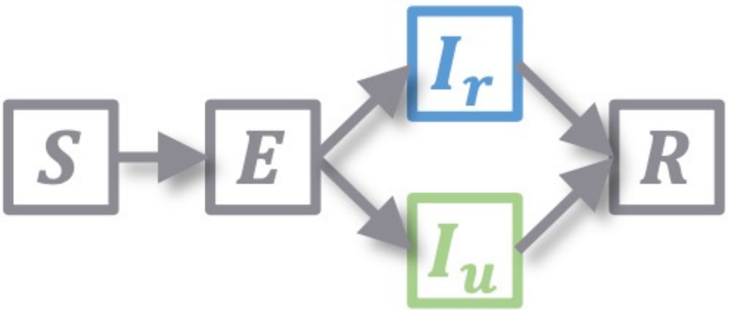
- **Epidemiological models**

Reported rate in epi models & estimation

- Many epi models have reported rate as a parameter
- Example:



$$
\begin{cases}
\frac{dS}{dt} = -\beta \frac{SI}{N} \\
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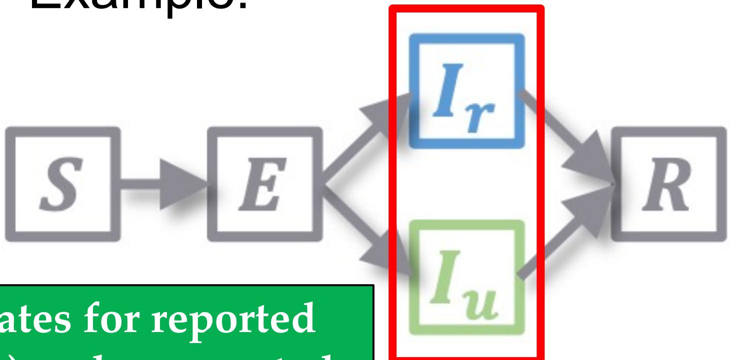

States for reported (I_r) and unreported cases (I_u)

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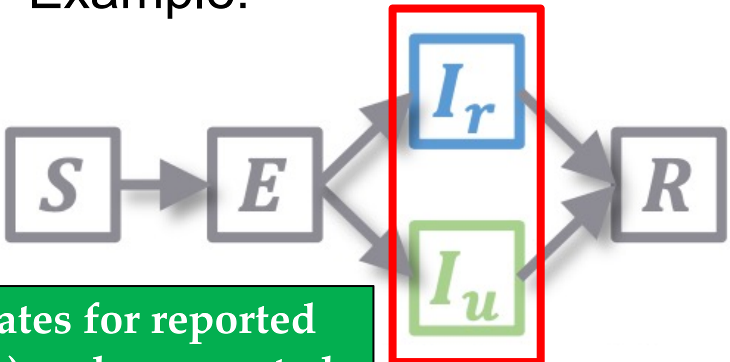

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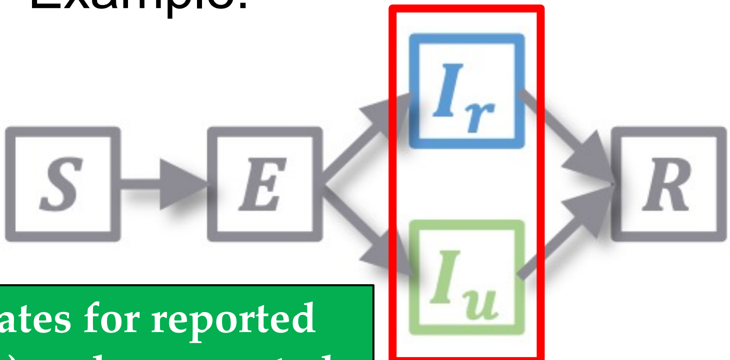

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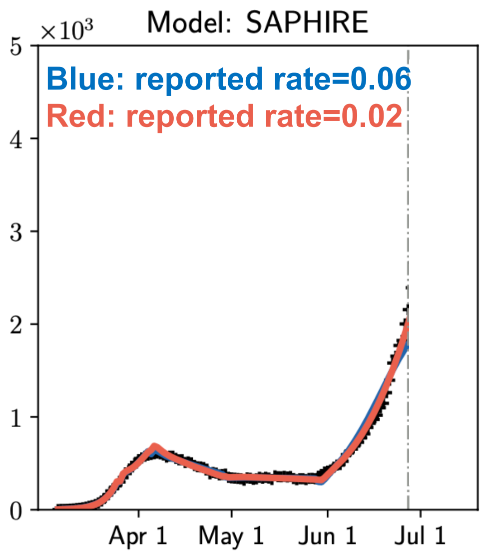
States for reported (I_r) and unreported cases (I_u)

$\alpha_{reported}$: reported rate

- Calibration: Fit I_r to reported cases to estimate the unknown parameters (including reported rate)

Sub-optimal calibration

- Parameter tuning is hard
  - Example: COVID-19 cases in Florida in 2020



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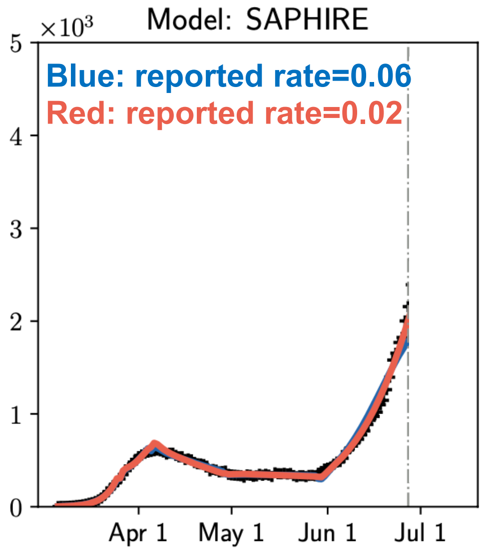

Both parameter fit the training data well.

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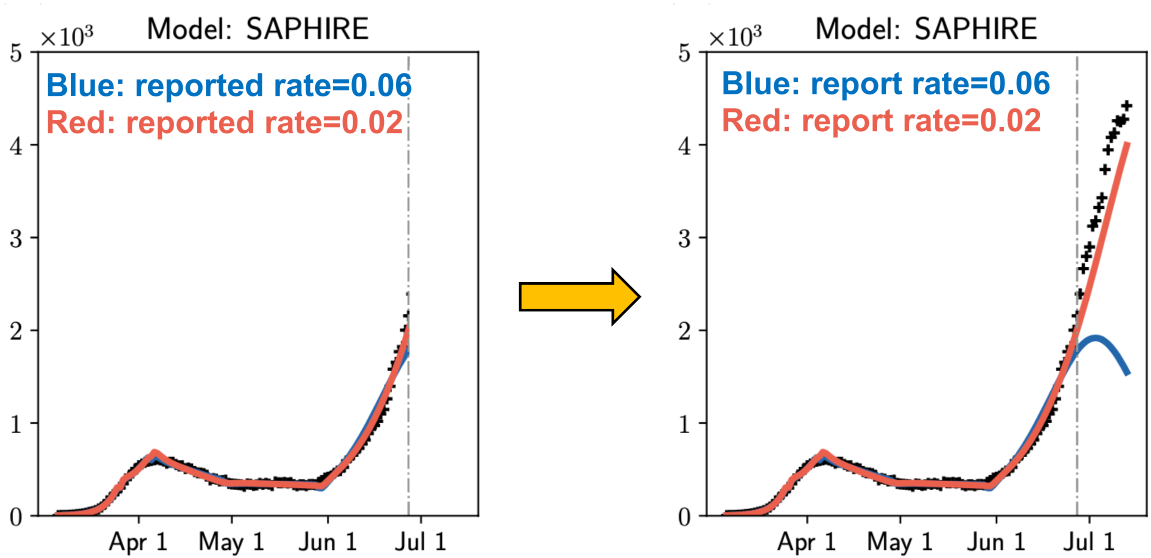

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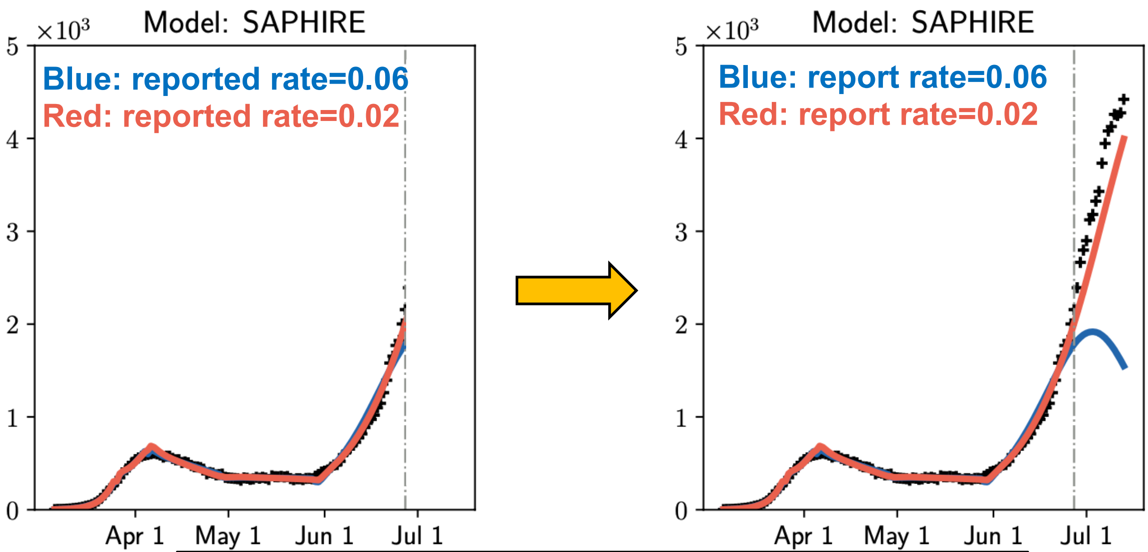

But their performance for forecasting are significantly different

Our goal

- Therefore, our goal is to estimate an **accurate** reported rate

Intuition

- We already know reported cases $D_{reported}$
- Imagine we are also given accurate values of total cases D
  - Then calibrating the model to $\left(D, D_{reported}\right)$ will lead to better fit of $D_{reported}$
  - We can also learn better parameters including $\alpha_{reported}$

- Hence, our problem can be stated as finding the D^* that fits the $D_{reported}$ best

Outline

- Introduction
- Two-part MDL: Sender-receiver framework
- MDLINFER: Information-theory based approach
- Experiment results
- Conclusions

Two-part MDL: Sender-receiver framework

- Two hypothetical actors: Sender S and Receiver R
  - Sender S wants to send the “DATA” to Receiver R using a good “ MODEL”
  - To measure the “good fit”, use the number of bits to encode DATA: $\mathrm{L(DATA|MODEL)} + \mathrm{L(MODEL)}$

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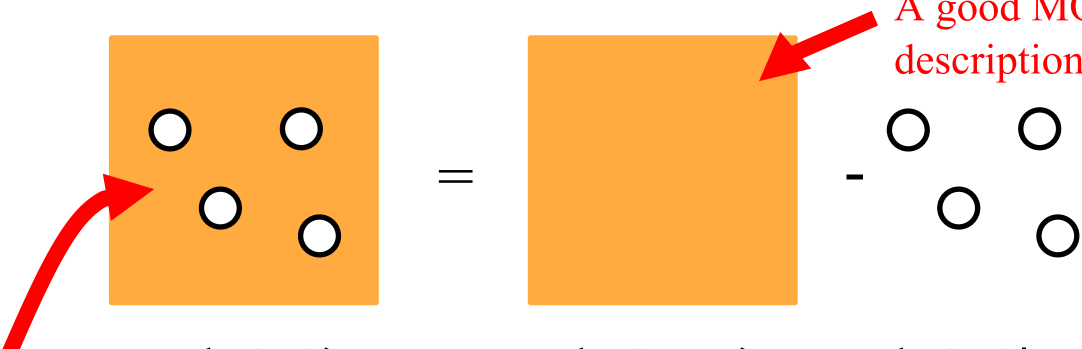
A good MODEL makes description easier!



Describe it directly is hard!

$$\mathrm{L(DATA) = L(MODEL) + L(DATA|MODEL)}$$

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Sender S searches for the best possible MODEL, which minimizes the overall cost of encoding and transmitting both the MODEL and the DATA given the MODEL.

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MDLINFER: MODEL and DATA

- Recall that our problem can be stated as finding the D^* that fits the $D_{reported}$ best

- We also have

$$D_{\mathrm{reported}} = \alpha_{\mathrm{reported}} \times D$$

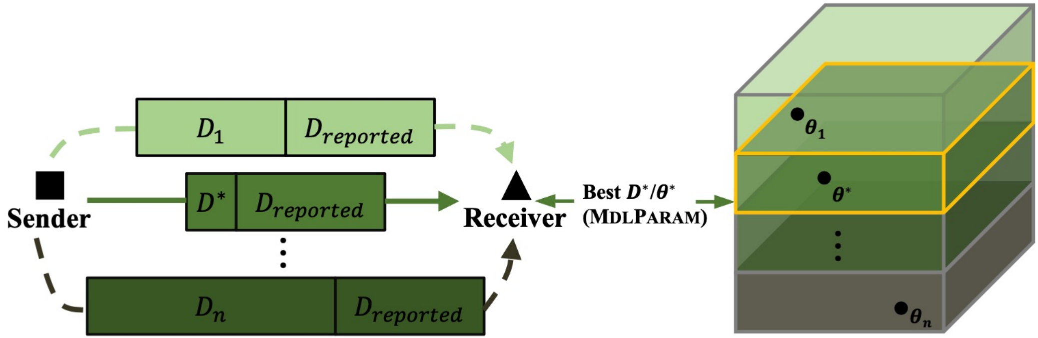
- Hence, we intuitively use
  - D_{reported} as DATA
  - D and $\alpha_{\mathrm{reported}}$ as MODEL

MDLINFER: Problem formulation

- Problem formulation

$$\boxed{D^* = \underset{D}{\operatorname{argmin}}\, L\left(D_{reported} \middle| D, \alpha_{\text{reported}}\right) + L(D, \alpha_{\text{reported}})}$$

- Here, $L(\cdot)$ denotes the number of bits for encoding



Use MDL Sender-Receiver framework to help find the latent variable D^* that explains $D_{reported}$ best.

The latent variable D^* helps find more accurate θ^* for the epidemiological model.

MDLINFER: Problem formulation

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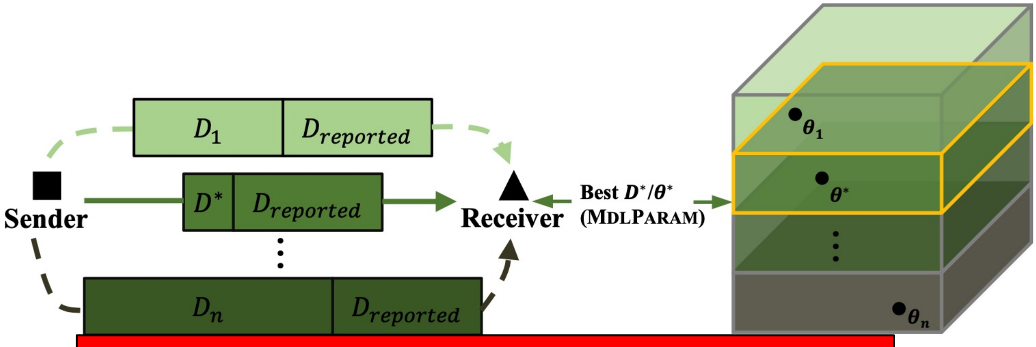

Use M... the lat... ...le D^* helps ...te θ^* for the ...model.

Intuitively, by trying different D, we do fine-grain search in the search space.

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Results

- More accurate estimation of total cases
  - Black: ground-truth total cases by serological studies [1]
  - Red for us, blue for current estimation methods

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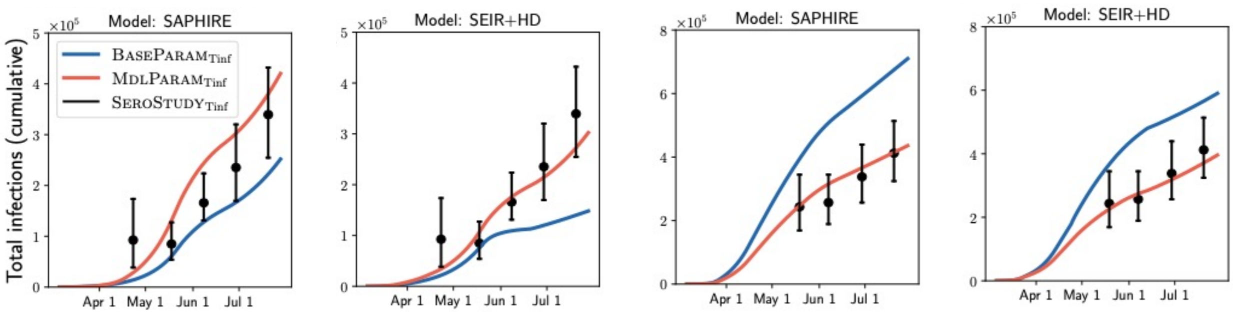
Minneapolis-Spring 2020 Philadelphia-Spring 2020



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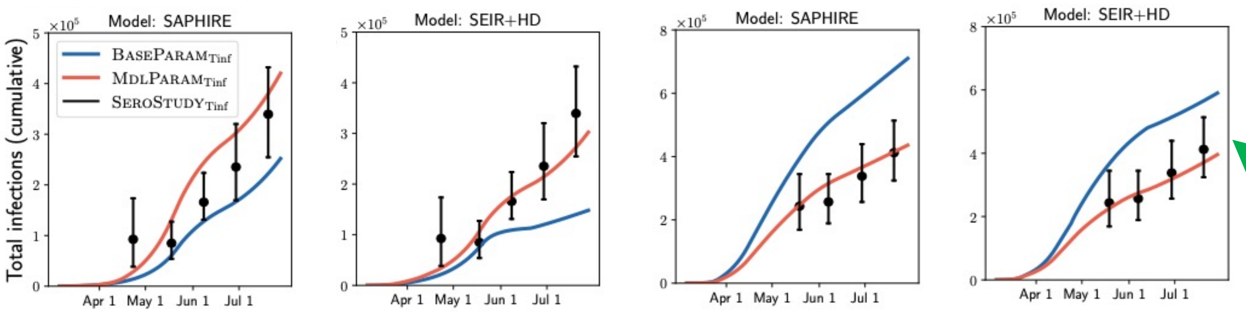

We show our performance on two different ODE models

Our method (red) always fit the serological studies well

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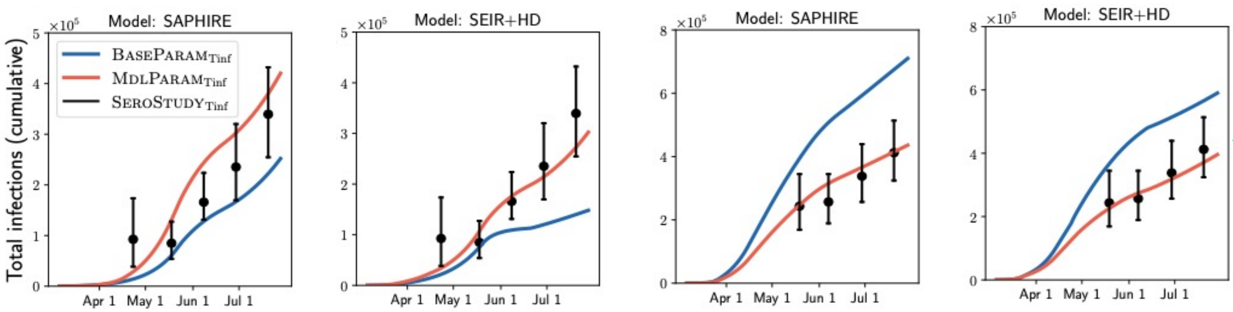

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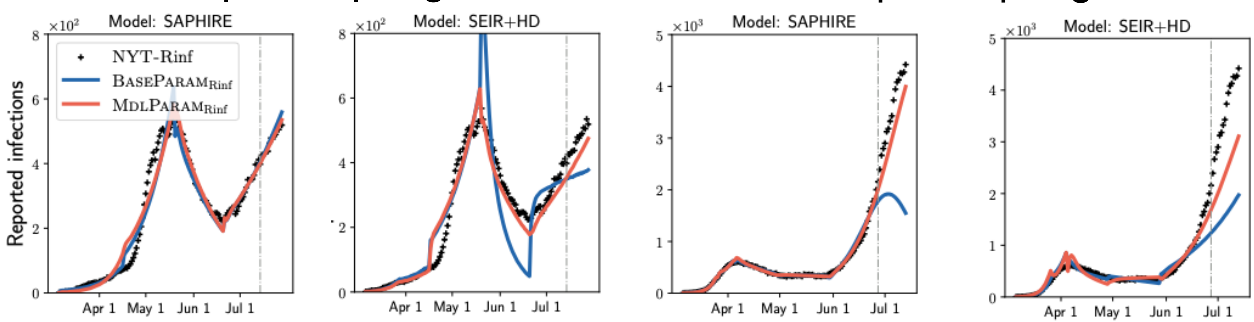

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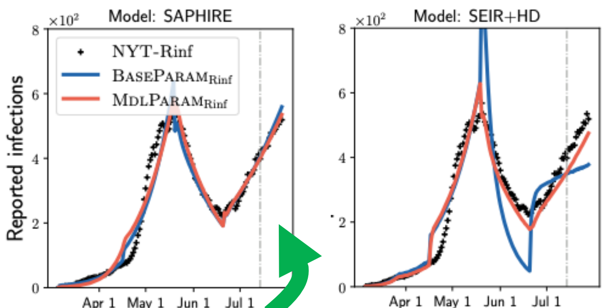

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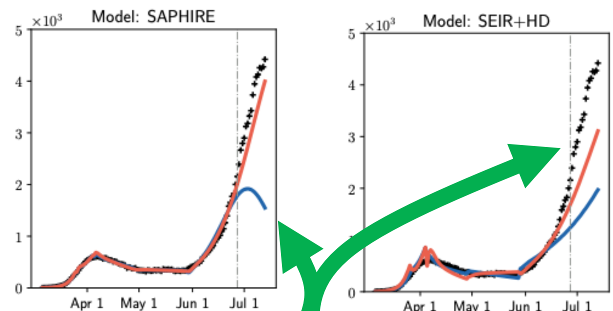

Grey dash line divides observed period (left) and forecast period (right).

Good performance in predicting future!

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More results for other locations in the paper

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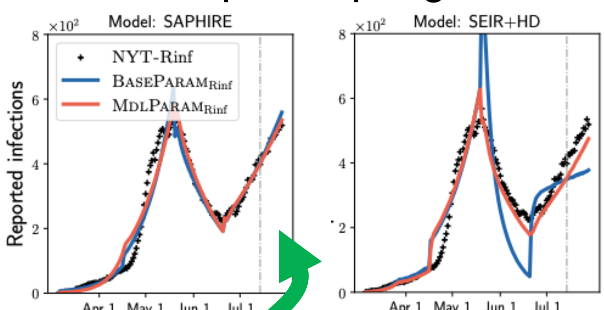

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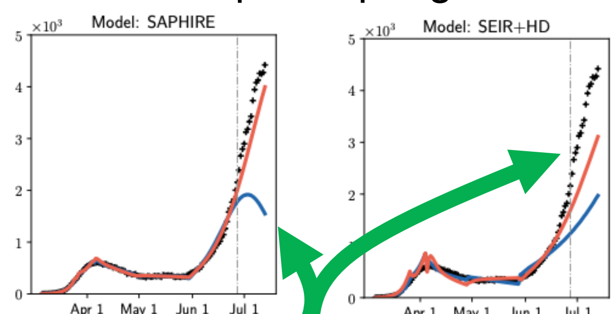

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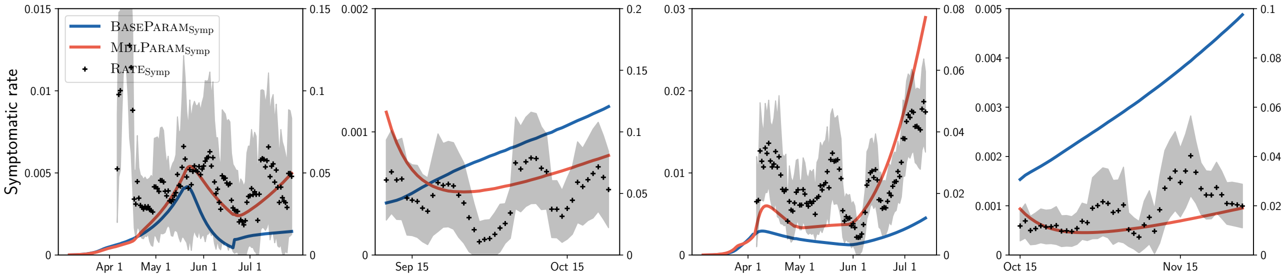

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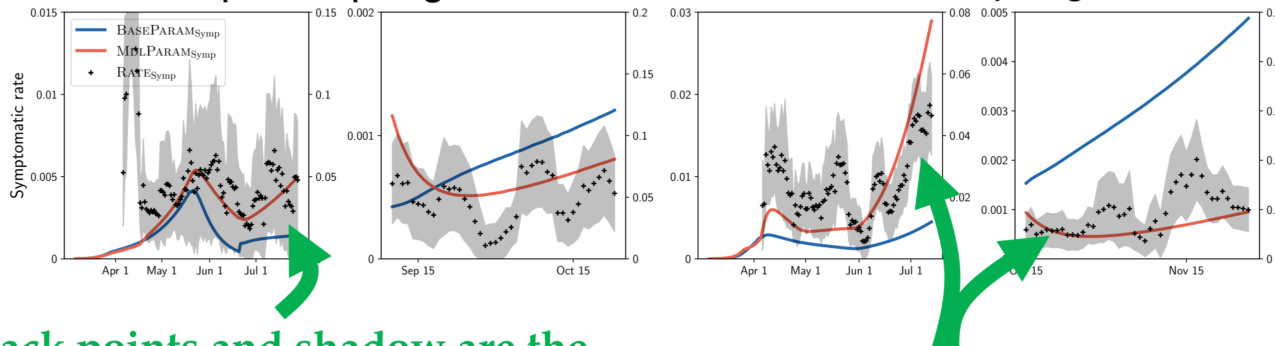

Black points and shadow are the point estimate and standard error

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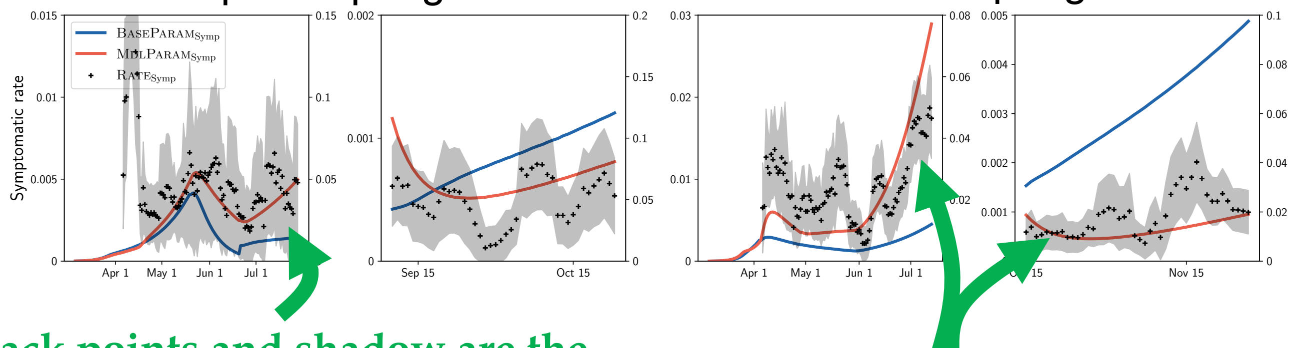

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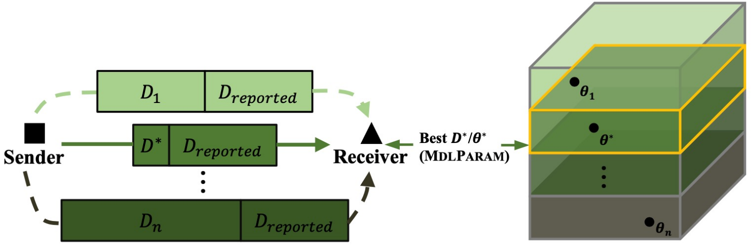
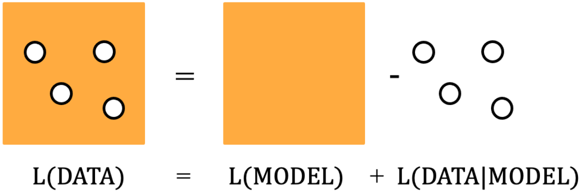
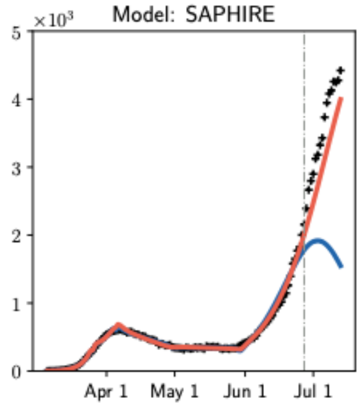


- We propose MDLINFER, a data-driven method to identify reported rate



- Leverage the information theory-based MDL framework

- Better performance in identifying total infections and forecasting future infections



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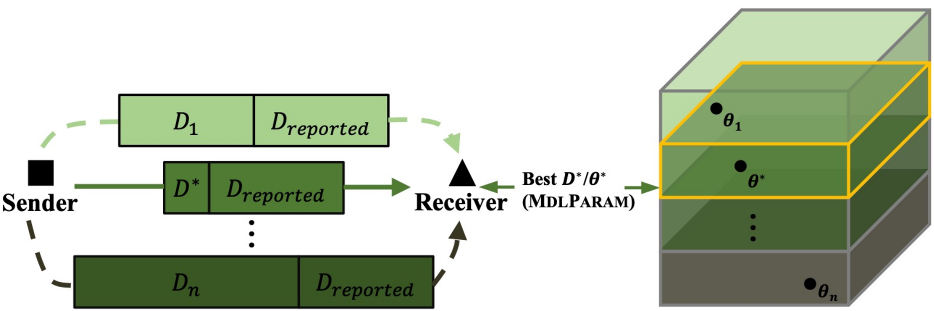
Thank you

- Code & papers available at: people.cs.vt.edu/jiamingcui/

Link to the paper:



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