NEAR-OPTIMAL SPECTRAL DISEASE MITIGATION IN HEALTHCARE FACILITIES

Masahiro Kiji, D. M. Hasibul Hasan, Alberto M. Segre, Sriram V. Pemmaraju, and **Bijaya Adhikari**

University of Iowa

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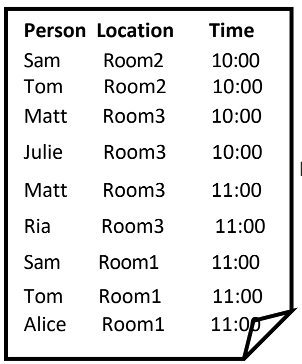
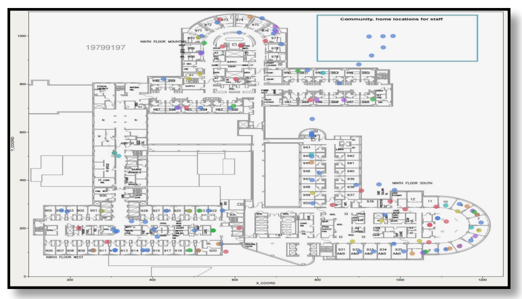
CONTACT NETWORKS IN HOSPITALS

- Temporally dynamic bipartite people-location networks

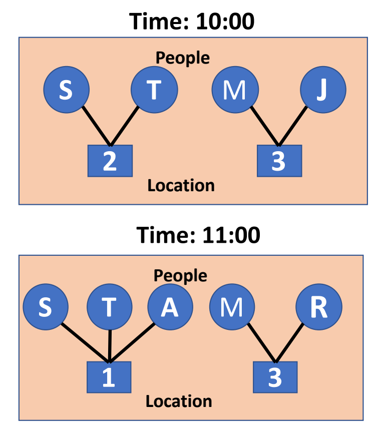


Person	Location	Time
Sam	Room2	10:00
Tom	Room2	10:00
Matt	Room3	10:00
Julie	Room3	10:00
Matt	Room3	11:00
Ria	Room3	11:00
Sam	Room1	11:00
Tom	Room1	11:00
Alice	Room1	11:00

Schedule logs



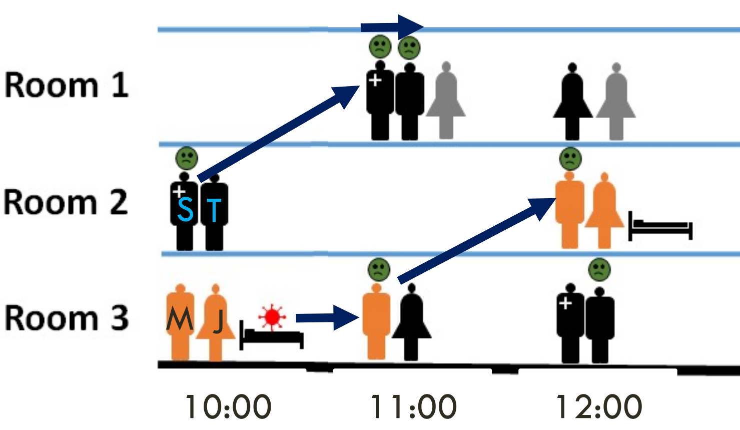
Spatial layout



High resolution
dynamic bipartite networks

PATHOGEN SPREAD IN HOSPITALS

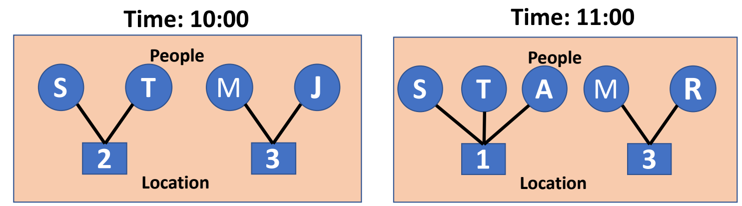
- Hospital Acquired Infections (HAI) spread in hospitals to/from healthcare workers and patients
- Examples: C. Diff, MRSA etc
- A major financial and health burden
- Bacterial load accumulates on surfaces
- Infects individuals

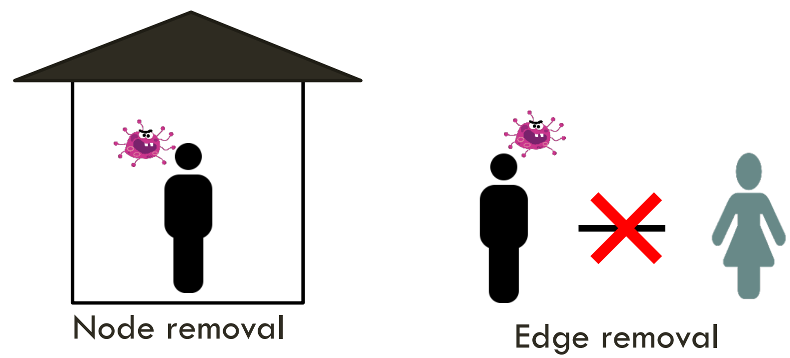


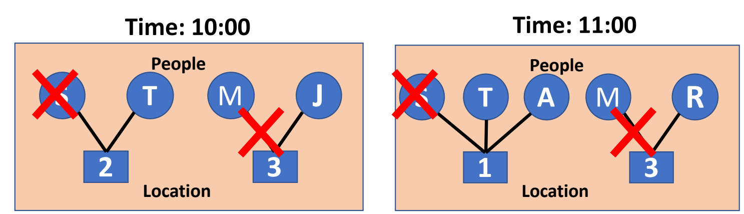
How to combat HAI spread?

INFORMAL PROBLEM STATEMENT

- **Given:** a sequence of mobility graphs
- **Determine:** top-k nodes/edges to remove
  - Node removal: quarantine
  - Edge removal: contact prevention
- **Such that:** the resulting graph is less vulnerable to outbreak







FORMULATION CHALLENGES

- Formulation challenge 1: How to define vulnerability?
  - How to characterize the disease spread?
  - How to quantify 'vulnerability'?

- Formulation challenge 2: How to formalize node/edge removals?
  - Node edges repeat over time
  - How to handle that?

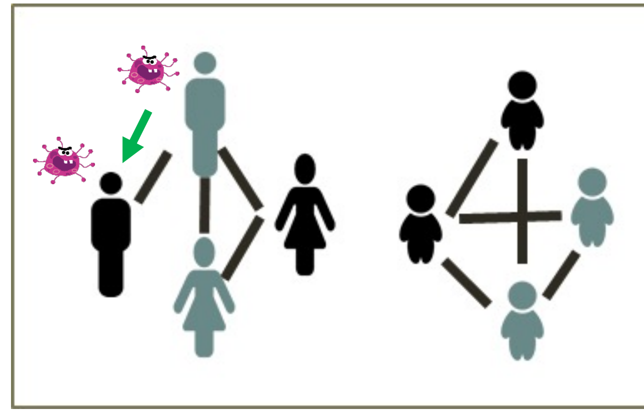
FC1: DISEASE SPREAD MODEL

SI MODEL

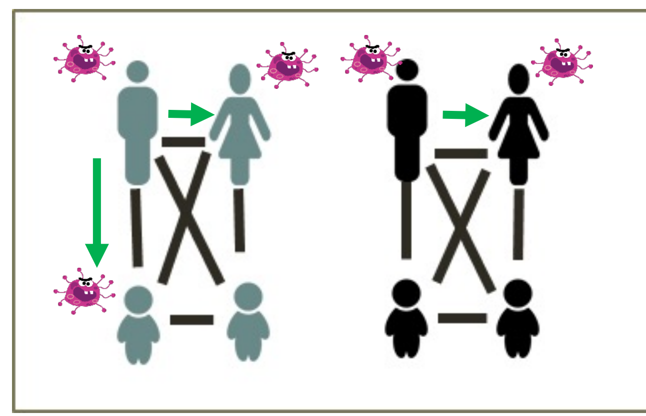
- "Infected" nodes infect "Susceptible" neighbors
- With probability given by edge-weight
- Once a node is infected, it remains infected
- Nodes get multiple chances to infect

IC MODEL

- Nodes get single chance to infect
- Nodes get cured in the next time-step
- Never become infected again



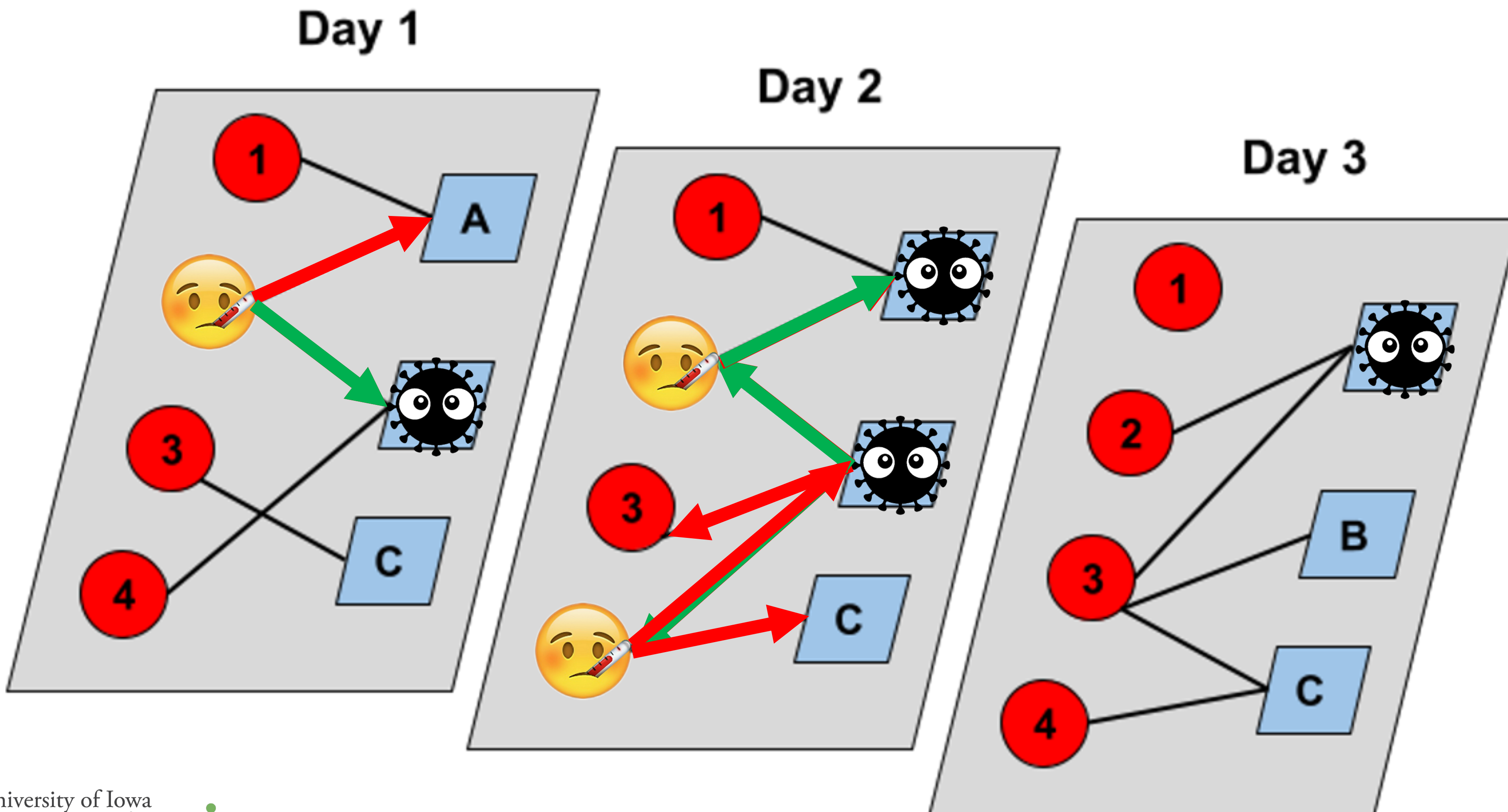
Day graph: Office/School



Night graph: Family

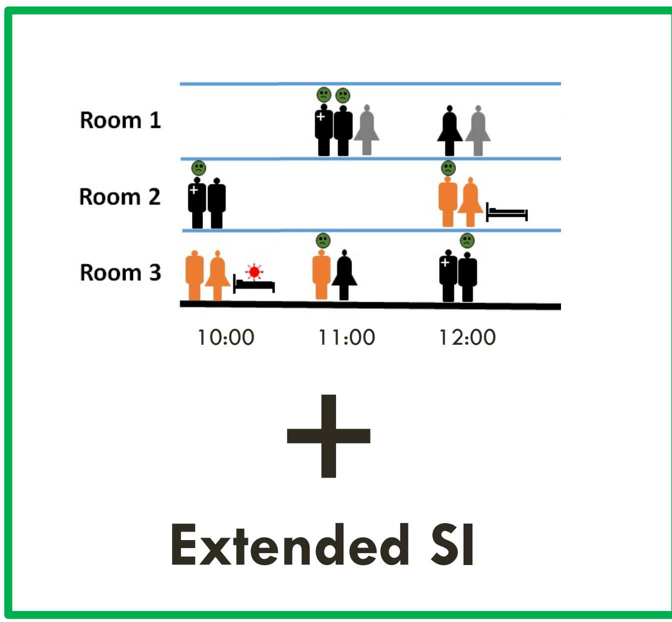
FC1: DISEASE SPREAD MODEL

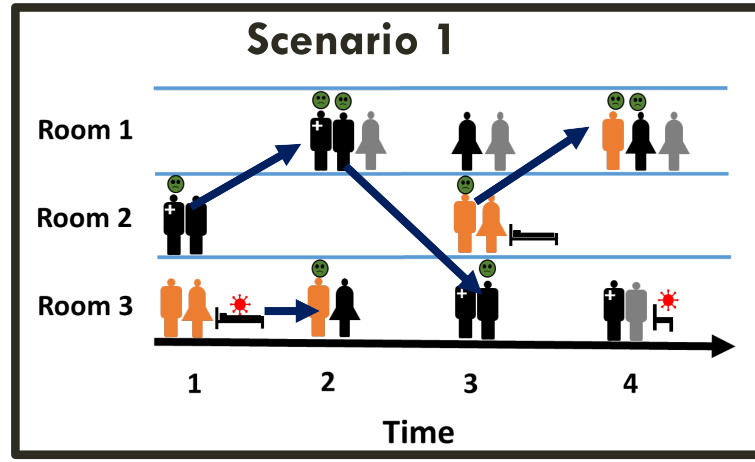
- Extended SI model for HAI

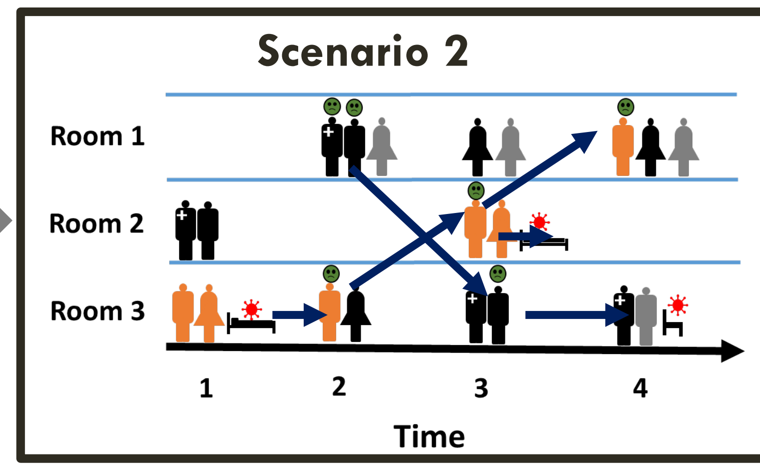


FC1:QUANTIFYING VULNERABILITY

- Use the calibrated SI model and the mobility log to produce possible outbreak scenarios







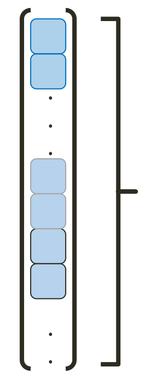
⋮

Problem: computationally too expensive

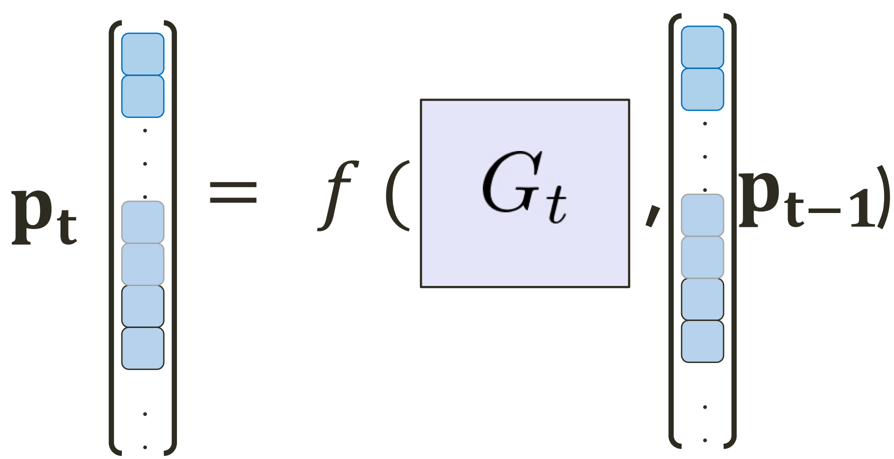
IDEA: LOOK FOR ASYMPTOTIC BEHAVIORS

- We are given a sequence of $\mathcal{G} = \{G_1, G_2, \ldots, G_T\}$ graphs
- Imagine a vector $\mathbf{p_t}$ which captures probability of infections
- Easy to see that $\mathbf{p_t}$ depends on $\mathbf{p_{t-1}}$ and G_t

$\mathbf{p_t}$



size |P| (number of people nodes in the graph)



$$\mathbf{p_t} = f\left(G_t, \mathbf{p_{t-1}}\right)$$

Non-linear dynamical system

EVOLUTION OF P

$$\mathbf{p_t} = f(G_t, \mathbf{p_{t-1}})$$

- $f(.)$ can be viewed as a discrete time NLDS

$$\mathbf{p_t}[v] = \mathbf{p_{t-1}}[v](1-\delta) + \beta_p\beta_l \sum_{l=1}^{L}\sum_{p=1}^{P} \mathbf{B_{t-1}}[l,u]\mathbf{B_{t-1}^{T}}[i,v]\mathbf{p_{t-1}}[u]$$

Probability that it is infected at time t-1 and does not recover

Probability that node v is infected at time t

Probability that v is infected by a location l which was infected by node u at t-1

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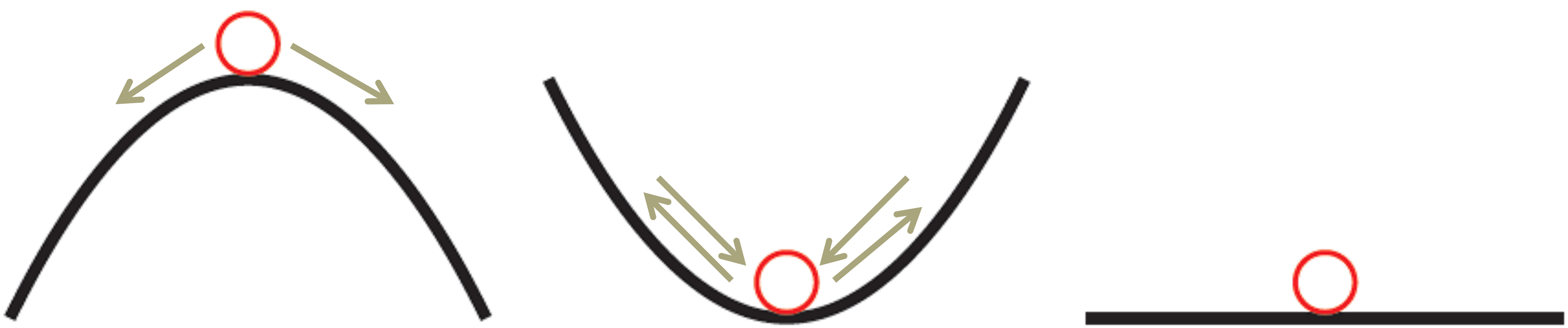
STABILITY OF A NLDS

THEOREM 3.1. (HIRSCH AND SMALE [1]) *The system given by a NDLS* $\mathbf{p}_t = g(\mathbf{p}_{t-1})$ *is asymptotically stable at an equilibrium point* $\mathbf{p}^*$ *(in our case the zero vector), if the eigenvalues of the Jacobian* $J = \nabla g(\mathbf{p}_{t-1})$ *are less than 1 in absolute value, where,*

$$J_{i,j} = \nabla g(\mathbf{p}_{t-1})_{i,j} = \frac{\partial p_{i,t}}{\partial p_{j,t-1}}\Big|_{\mathbf{p}_t=\mathbf{p}^*}$$



(A) Unstable (B) Stable (C) Neutral (at threshold)

EPIDEMIC THRESHOLD OF G

- Jacobian of $f()$ at $\mathbf{p_t} = \mathbf{0}$

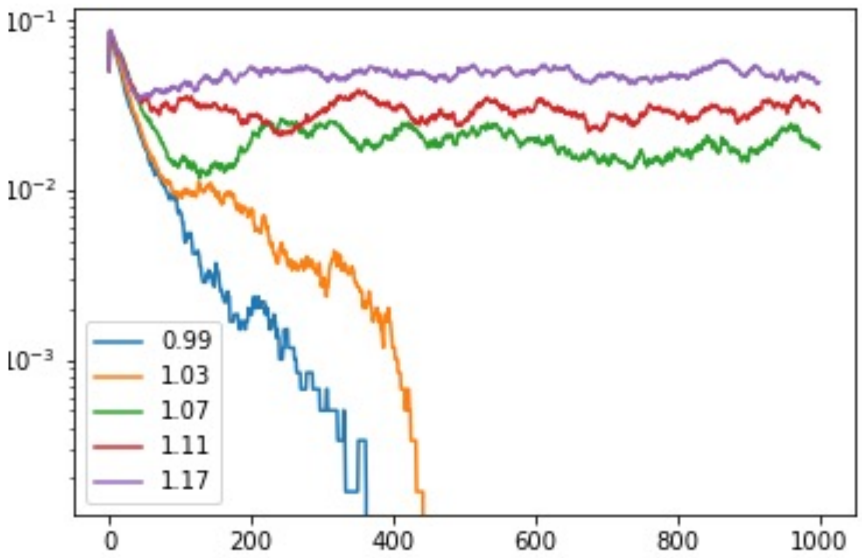
$$\mathbf{S} = \prod_{t=1}^{\tau} (1 - \delta_p)\mathbf{I} + \beta_p \beta_l \mathbf{B}_t \mathbf{B}^{\intercal}_{(t \bmod \tau)+1}$$

Disease Parameters (δ_p, $\beta_p \beta_l$) **Adjacency matrices** ($\mathbf{B}_t \mathbf{B}^{\intercal}_{(t \bmod \tau)+1}$)

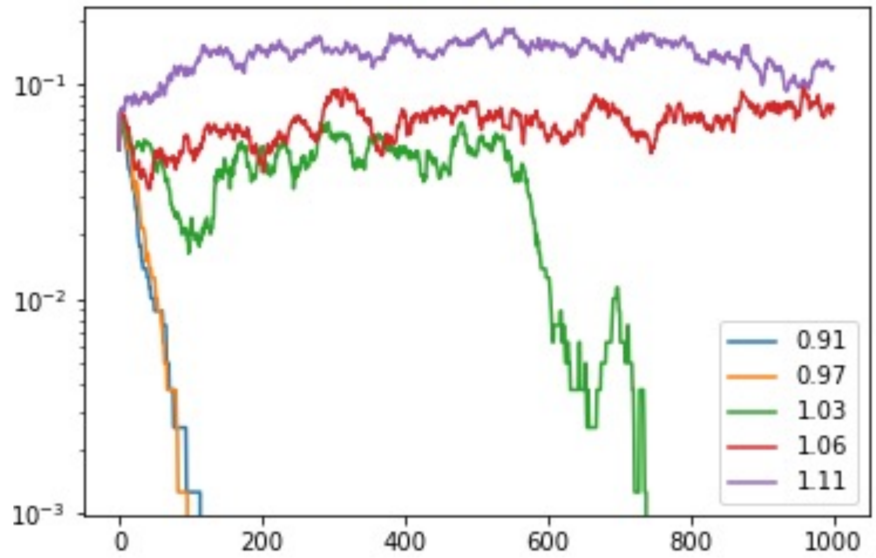
- Therefore,

THEOREM 3.1. *If* $\lambda_{\mathbf{S}} < 1$, *then* $\mathbf{p}_t$ *is asymptotically stable at* $\mathbf{0}$.

EPIDEMIC THRESHOLD VALIDATION



UIHC



Commercial HC 1

Infection dies out when $\lambda_S \ll 1$, survives for few time-stamps when $\lambda_S \approx 1$, and continues on when $\lambda_S \gg 1$

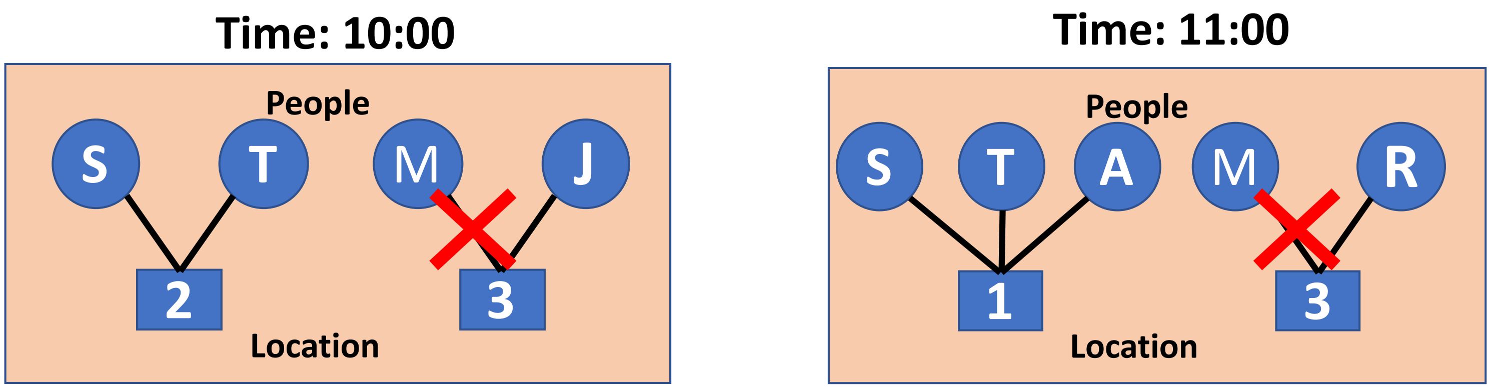
FORMULATION CHALLENGES

- ☑ Formulation challenge 1: How to define vulnerability?
  - **How to characterize the disease spread?**
  - **How to quantify ‘vulnerability’?**

- ☐ Formulation challenge 2: How to formalize node/edge removals?
  - Node edges repeat over time
  - How to handle that?

FC2: EDGE REMOVAL

- When we remove an edge from $G = g_1, g_2 \ \ldots, g_\tau$, we remove it from all time-stamps



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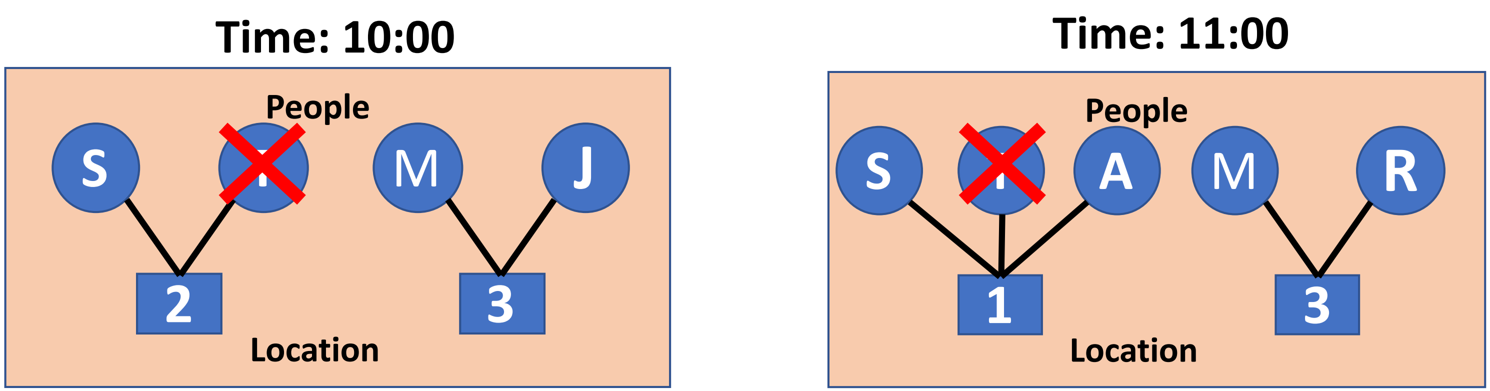


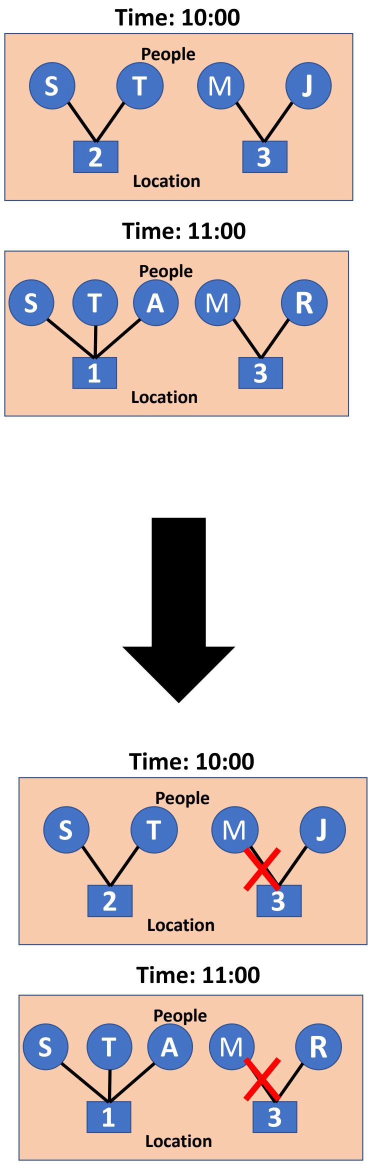
FORMAL PROBLEM DEFINITION

- **Given:** A temporal bipartite network $\mathcal{G} = \{G_1, G_2, \ldots, G_T\}$ and $\alpha_E \in (0, 1]$

- **Find:** A smaller graph

$$\mathcal{G}^* = \{G_1^*, G_2^*, \ldots, G_T^*, \}$$

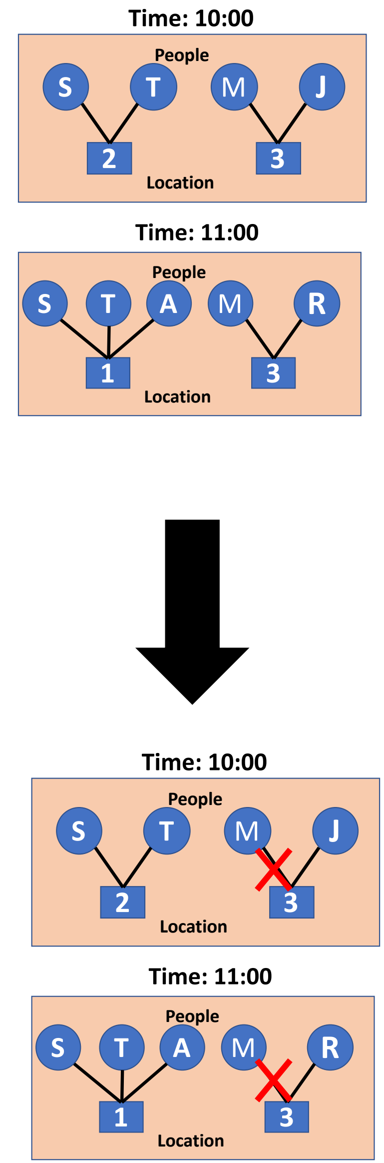
- **Such that:** $1 - \alpha_E$ edges are removed at maximum and $\mathcal{G}^* = \arg\min_{G'} \lambda_{\mathbf{S}'}$



HOW TO SOLVE THE PROBLEM

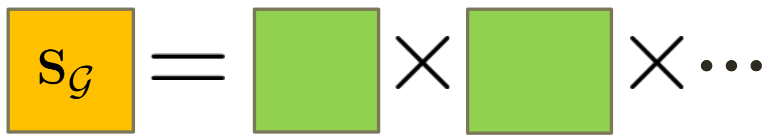
NAÏVE- Top-k

1. For each edge in the graph
   1. Remove it
   2. Compute Δ-scores = the drop in the largest eigenvalue of S
2. Sort the edges in decreasing order of Δ-scores
3. Until the graph is small enough do
   1. Delete the best possible edge

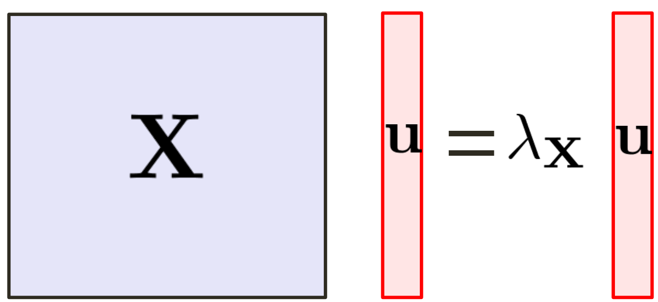


PROBLEM WITH THE NAÏVE APPROACH

- Expensive to compute S

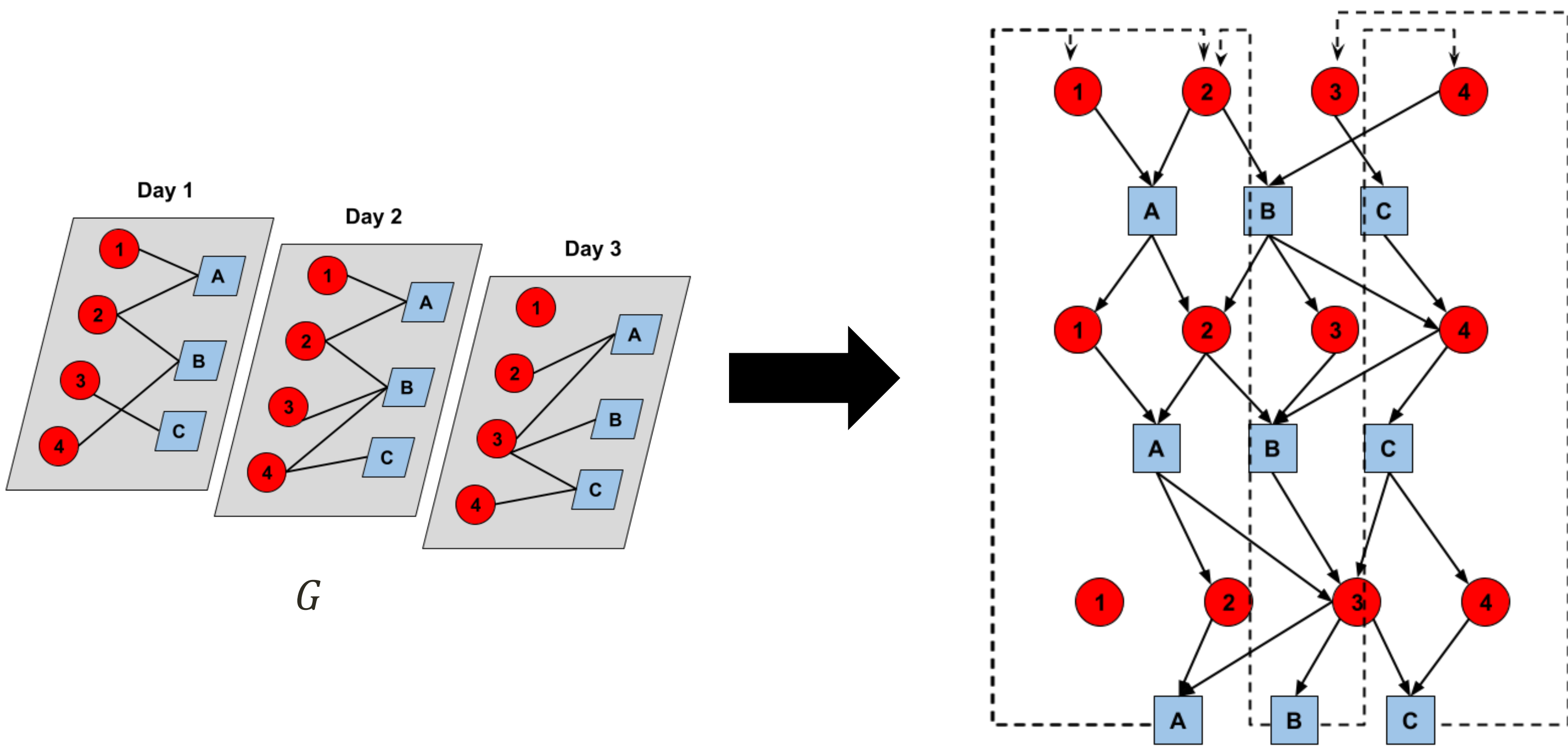


- Expensive to repeatedly compute largest eigenvalue



HOW TO AVOID COMPUTING S AND λ

- First create a static graph F from G



G F

WHAT IS SO SPECIAL ABOUT F?

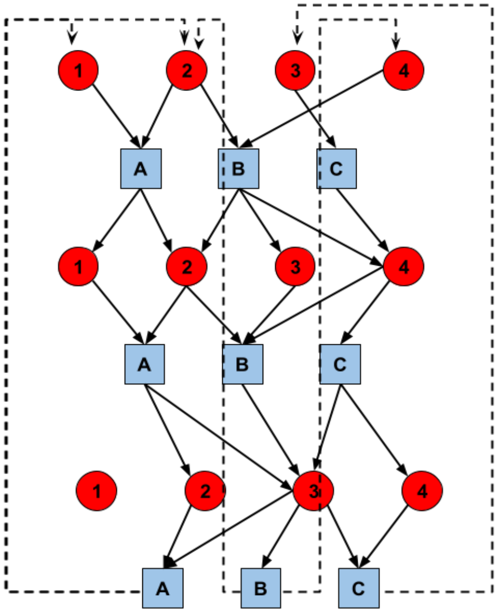
- Turns out the number of closed walks in F and the sum of eigenvalues of S are very closely related.

$$\sum_{w \in W(F)} nodes(w) = \frac{k}{(\beta_p \beta_l)^{k/2}} \sum_{i}^{n} (\lambda_i(\mathbf{S_{IC}}))^{k/2\tau}$$

Sum of unique nodes in each closed walks in F

Hyper and disease parameters

Sum of eigenvalues



F

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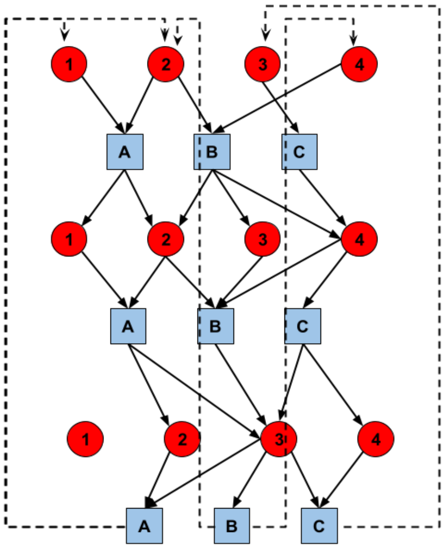
A NEAR OPTIMAL ALGORITHM

Temporal Edge Cover

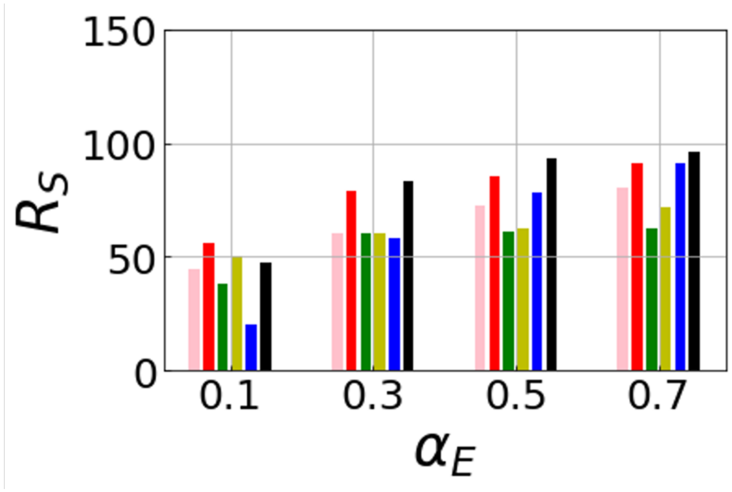
1. Compute F from G
2. While not enough edges are removed
   1. Compute the number of closed walks each edge participates in
   2. Remove the edge that is in the most number of closed walks



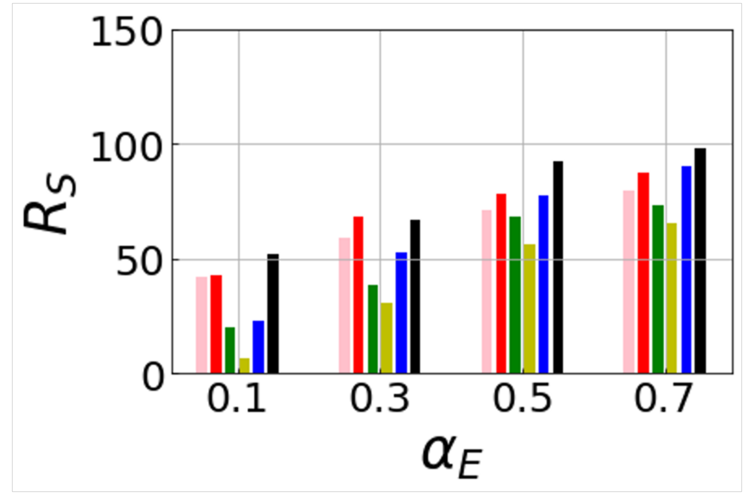
F

Provably near optimal.

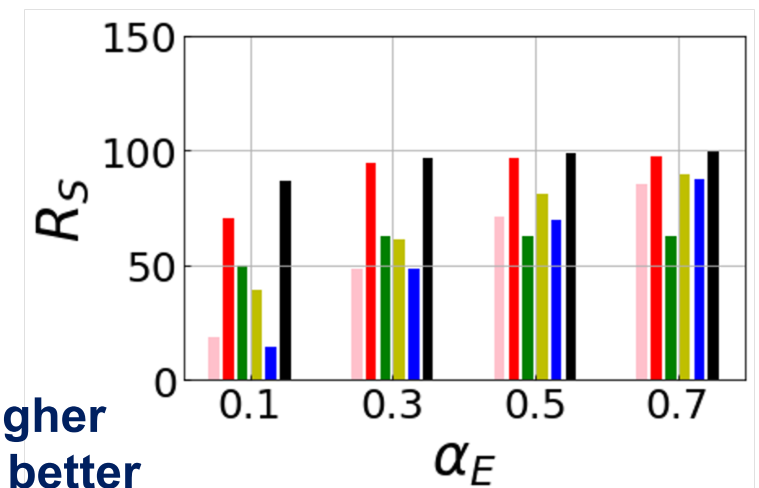
PERFORMANCE: EDGE DELETION



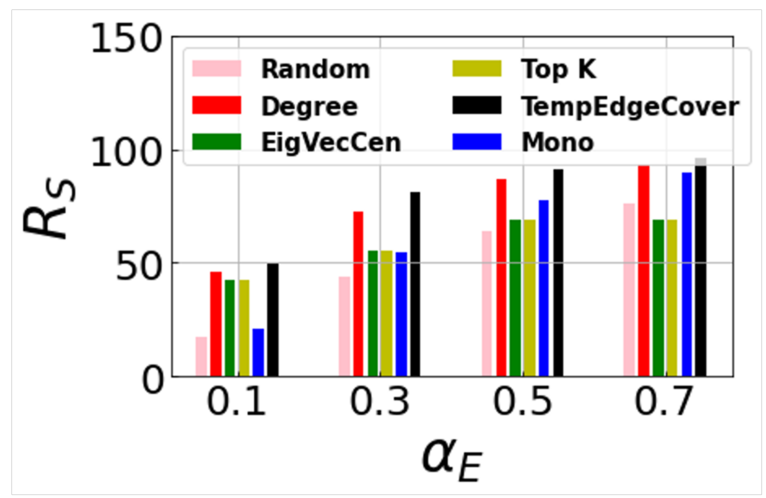
Commercial HC 1



Commercial HC 2



Carilion



UIHC

Higher is better

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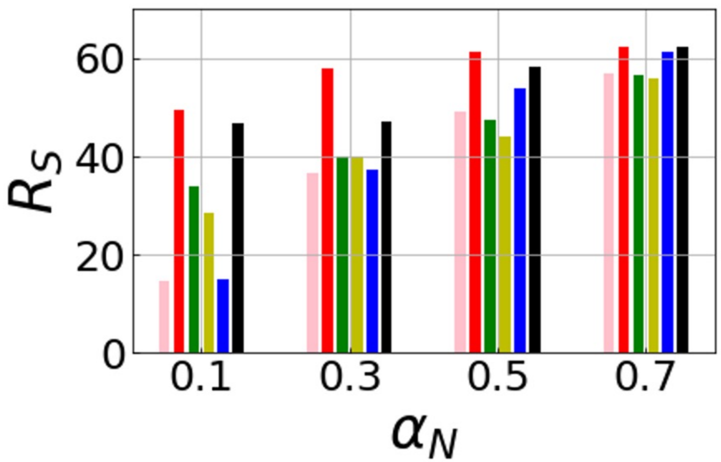
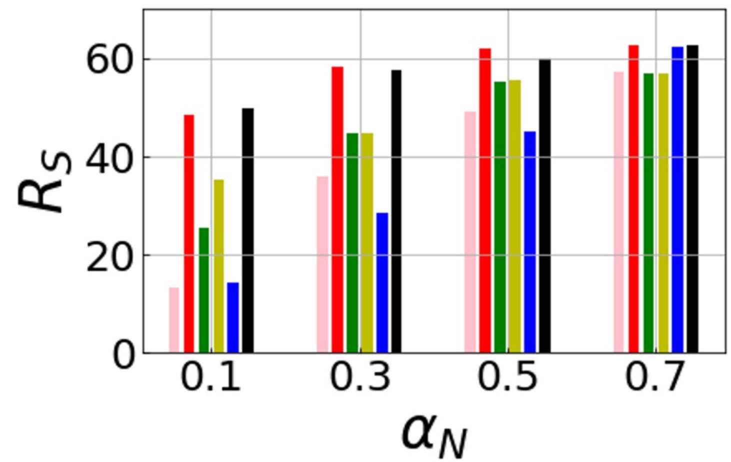
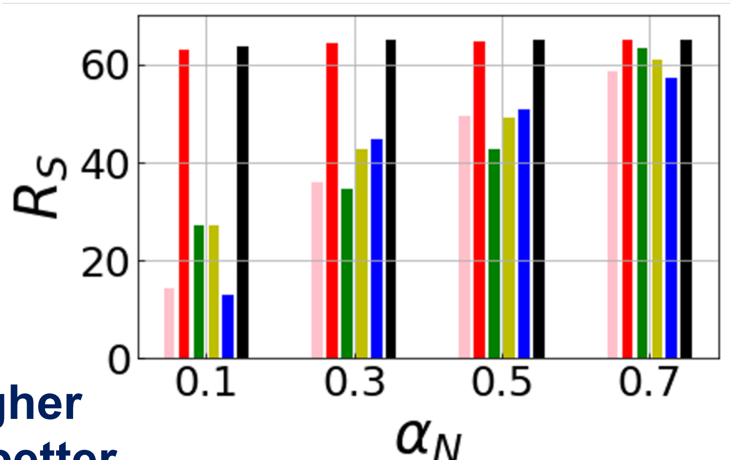
PERFORMANCE: NODE DELETION



Commercial HC 1



Commercial HC 2



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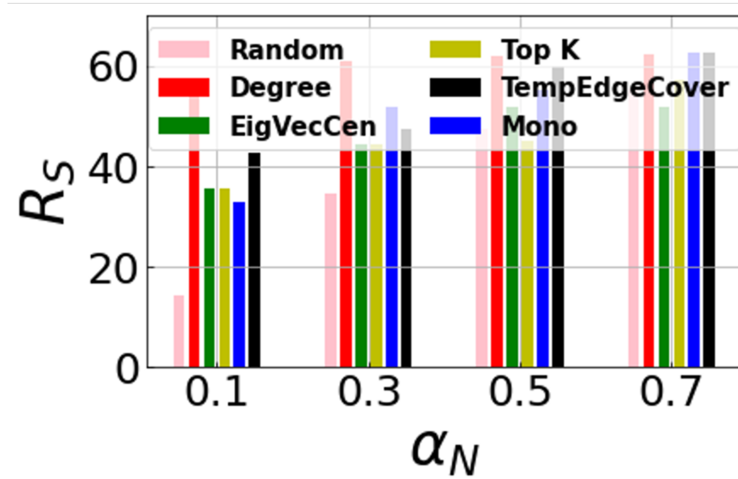

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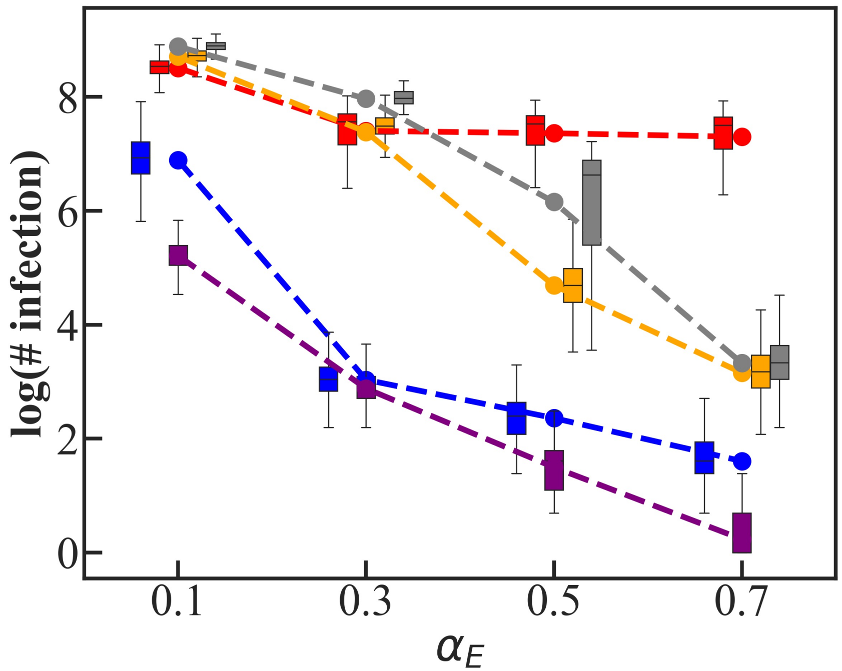
Higher is better

INFECTION CONTROL: BSIS



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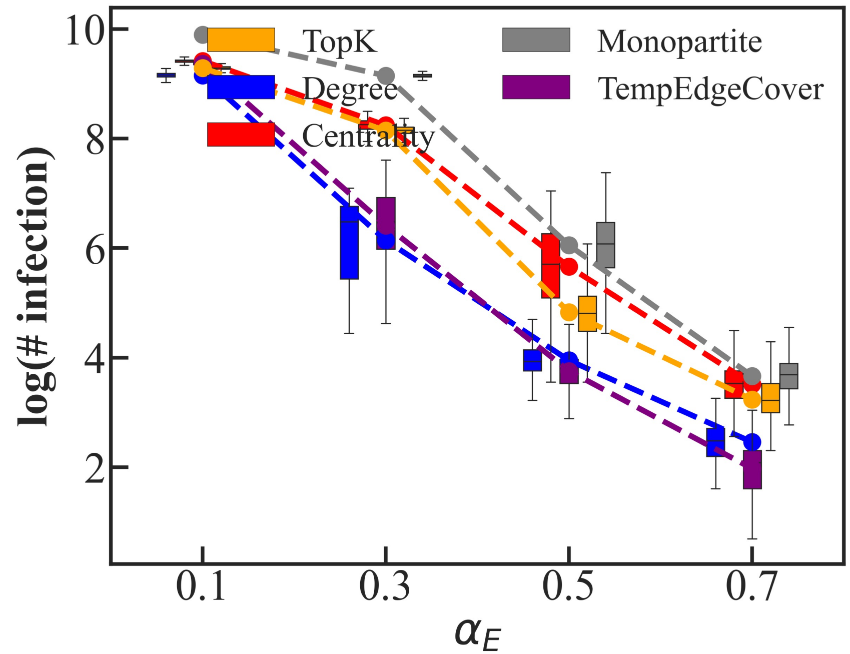

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Our approach leads to improved infection control.

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