Improved Dropout for Shallow and Deep Learning

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1 Introduction and Problem Setup

2 Improved Dropout for Shallow Learning

3 Improved Dropout for Deep Learning

4 Experimental Results

5 Conclusion
Outline

1. Introduction and Problem Setup
2. Improved Dropout for Shallow Learning
3. Improved Dropout for Deep Learning
4. Experimental Results
5. Conclusion
The success of deep learning

- Image Classification
The success of deep learning

- Image Classification
The success of deep learning

- Image Classification
  - Cat
  - Dog
  - Goldfinch
The success of deep learning

- Image Classification

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Deep Neural Network

The classical example: AlexNet [A Krizhevsky, et al., 2012]
Dropout Layer:

- Dropout Layer: Uniformly at randomly drop out features.
Dropout Layer

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- Is uniformly dropout optimal?
Dropout Layer

- Dropout Layer: Uniformly at randomly drop out features.

- Is uniformly dropout optimal?
  - Answered the above question in this work.
Improved Dropout

- Dropping out the output of the neuron based on multinomial distribution computed from the training data.

**Figure:** Evolutioanal dropout vs standard dropout on CIFAR100 datasets for deep learning
Problem Setup

Let \((x, y)\) denote a feature vector and a label, where \(x \in \mathbb{R}^d\) and \(y \in \mathcal{Y}\).
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- Denote by \(\mathcal{P}\) the joint distribution of \((x, y)\) and by \(\mathcal{D}\) the marginal distribution of \(x\).
- The goal is to learn a linear prediction function \((f(x) = w^\top x)\) that minimizes the expected risk (considering loss function \(\ell(\cdot, y)\)):

\[
\min_{w \in \mathbb{R}^d} \mathcal{L}(w) \triangleq \mathbb{E}_\mathcal{P}[\ell(w^\top x, y)]
\]  

(1)
Problem Setup

- Denote by \( \epsilon \sim \mathcal{M} \) a dropout noise vector of dimension \( d \).
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- Denote by $\hat{P}$ the joint distribution of the new data $(\hat{x}, y)$ and by $\hat{D}$ the marginal distribution of $\hat{x}$.
- With the corrupted data, the risk minimization becomes

$$
\min_{w \in \mathbb{R}^d} \hat{\mathcal{L}}(w) \triangleq \mathbb{E}_{\hat{P}}[\ell(w^\top (x \circ \epsilon), y)] \quad (2)
$$
Multinomial Dropout

Definition 1

A multinomial dropout is defined as \( \hat{x} = x \circ \epsilon \), where \( \epsilon_i = \frac{m_i}{kp_i}, i \in [d] \) and \( \{m_1, \ldots, m_d\} \) follow a multinomial distribution \( \text{Mult}(p_1, \ldots, p_d; k) \) with \( \sum_{i=1}^{d} p_i = 1 \) and \( p_i \geq 0 \).
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- Ability of using non-uniformly sampling probabilities for different features.
- Easy to control the level of dropout by varying the value of \( k \).
Multinomial Dropout

- Dropout is a data-dependent regularizer.
Multinomial Dropout

- Dropout is a data-dependent regularizer.

**Proposition 1**

If $\ell(z, y) = \log(1 + \exp(-yz))$, then

$$E_{\hat{P}}[\ell(w^\top \hat{x}, y)] = E_P[\ell(w^\top x, y)] + R_{D,M}(w)$$

where $M$ denotes the distribution of $\epsilon$ and

$$R_{D,M}(w) = E_{D,M} \left[ \log \frac{\exp(w^\top \frac{x \circ \epsilon}{2}) + \exp(-w^\top \frac{x \circ \epsilon}{2})}{\exp(w^\top \frac{x}{2}) + \exp(-w^\top \frac{x}{2})} \right].$$
Learning with Multinomial Dropout

Give the initial solution $w_1$.

Update the model at $t$th iteration:

$$w_{t+1} = w_t - \eta_t \nabla \ell(w_t^\top(x_t \circ \epsilon_t), y_t) \quad (3)$$

Output the final solution:

$$\hat{w}_n = \frac{1}{n} \sum_{t=1}^{n} w_t$$
Learning with Multinomial Dropout

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Learning with Multinomial Dropout
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Theorem 1:

Let $\mathcal{L}(\mathbf{w})$ be the expected risk of $\mathbf{w}$ defined in (1). Assume $\mathbb{E}_{\mathcal{D}}[\|x \circ \epsilon\|^2_2] \leq B^2$ and $\ell(z, y)$ is convex and $G$-Lipschitz continuous. For any $\|\mathbf{w}_*\|_2 \leq r$, by appropriately choosing $\eta$, we can have

$$\mathbb{E}[\mathcal{L}(\hat{\mathbf{w}}_n) + R_{\mathcal{D},\mathcal{M}}(\hat{\mathbf{w}}_n)] \leq \mathcal{L}(\mathbf{w}_*) + R_{\mathcal{D},\mathcal{M}}(\mathbf{w}_*) + \frac{GBr}{\sqrt{n}}$$

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$$E[\mathcal{L}(\hat{w}_n) + R_{D,\mathcal{M}}(\hat{w}_n)] \leq \mathcal{L}(w_\star) + R_{D,\mathcal{M}}(w_\star) + \frac{GBr}{\sqrt{n}}$$

How to prove the above theorem?

- Standard SGD analysis.
Improved dropout for Shallow Learning

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How to prove the above theorem?
- Standard SGD analysis.
- Dropout is a data-dependent regularizer.
Improved dropout for Shallow Learning

- Minimizing the term $E_D[\|x \circ \epsilon\|^2_2]$ and the relaxed upper bound of term $R_D,M(w_*)$ yields the optimal sampling probabilities:

$$p_i^* = \frac{\sqrt{E_D[x_i^2]}}{\sum_{j=1}^{d} \sqrt{E_D[x_j^2]}}, \quad i = 1, \ldots, d$$

(4)
Improved dropout for Shallow Learning

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- Can we compute the above probability for dropout?
  - \(x\)
Improved dropout for Shallow Learning

Practically, we use the empirical second-order statistics to compute the probabilities:

\[ p_i = \frac{\sqrt{\frac{1}{n} \sum_{j=1}^{n} [x_j^2]_i}}{\sum_{i' = 1}^{d} \sqrt{\frac{1}{n} \sum_{j=1}^{n} [x_j^2]_{i'}}}, \quad i = 1, \ldots, d \]  

(5)
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Could we directly use the above idea to Deep Learning?
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\( \times \)

Why not?
Could we directly use the above idea to Deep Learning?

- \( \times \)

Why not?

- Too expensive to compute dropout probability from all examples.
Could we directly use the above idea to Deep Learning?

\( \times \)

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Too expensive to compute dropout probability from all examples.

How to address this issue?
Could we directly use the above idea to Deep Learning?

×

Why not?

Too expensive to compute dropout probability from all examples.

How to address this issue?

Use a mini-batch of examples to calculate the dropout probability.
Let $X^l = (x^l_1, \ldots, x^l_m)$ denote the outputs of the $l^{th}$ layer for a mini-batch of $m$ examples, calculate the probabilities for dropout by

$$p^l_i = \frac{\sqrt{\frac{1}{m} \sum_{j=1}^{m} [x^l_{ij}]^2}}{\sum_{i'=1}^{d} \sqrt{\frac{1}{m} \sum_{j=1}^{m} [x^l_{ij'}]^2}}, \quad i = 1, \ldots, d$$

(6)
Evolutional Dropout for Deep Learning

**Input:** a batch of outputs of a layer: \( X^l = (x^l_1, \ldots, x^l_m) \) and dropout level parameter \( k \in [0, d] \)

**Output:** \( \hat{X}^l = X^l \circ \Sigma^l \)

Compute sampling probabilities by (6)

For \( j = 1, \ldots, m \)

Sample \( m^l_j \sim Mult(p^l_1, \ldots, p^l_d; k) \)

Construct \( \epsilon^l_j = \frac{m^l_j}{kp^l_j} \in \mathbb{R}^d \), where \( p^l = (p^l_1, \ldots, p^l_d)^T \)

Let \( \Sigma^l = (\epsilon^l_1, \ldots, \epsilon^l_m) \) and compute \( \hat{X}^l = X^l \circ \Sigma^l \)

**Figure:** Evolutional Dropout applied to a layer over a mini-batch
Introduction and Problem Setup

Improved Dropout for Shallow Learning

Improved Dropout for Deep Learning

Experimental Results

Conclusion
Experimental Results for Shallow Learning

Training/test error between standard and improved dropout

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**Figure:** data-dependent dropout vs. standard dropout on three datasets (real-sim, news20 and RCV1) for logistic regression
Experimental Results for Deep Learning

- Implemented in CudaConvNet Library.
Experimental Results for Deep Learning

- Implemented in CudaConvNet Library.
- Using four benchmark datasets: MNIST, SVHN, CIFAR10, CIFAR100.
Implemented in CudaConvNet Library.

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Different neural network structures from the existing literatures.
Experimental Results for Deep Learning

- Implemented in CudaConvNet Library.
- Using four benchmark datasets: MNIST, SVHN, CIFAR10, CIFAR100.
- Different neural network structures from the existing literatures.
- Training strategy.
Figure: Evolutonal dropout vs. standard dropout on four benchmark datasets (MNIST, SVHN, CIFAR-10 and CIFAR-100) for deep learning
Experimental Results for Deep Learning

Compared to Batch Normalization

Figure: Evolutional dropout vs BN on CIFAR-10.
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- Proposed a multinomial dropout for shallow learning.
- Demonstrated that this proposed distribution-dependent dropout leads to a faster convergence and a smaller generalization error through the risk bound analysis.
- Proposed an efficient evolitional dropout for deep learning.
- Justified the proposed dropouts for both shallow and deep learning empirically.
Question?