

# CS:4420 Artificial Intelligence

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## Beyond Classical Search

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# Readings

- Chap. 4 of [Russell and Norvig, 3rd Edition]

# Beyond Classical Search

We have seen methods that systematically explore the search space, possibly using principled pruning (e.g., A\*)

The best of these methods can currently handle search spaces of up to  $10^{100}$  states /  $\sim 1,000$  binary variables (ballpark figure)

What if we have much larger search spaces?

Search spaces for some real-world problems may be much larger e.g.,  $10^{30,000}$  states as in certain reasoning and planning tasks

Some of these problems can be solved by **Iterative Improvement Methods**

# Iterative Improvement Methods

In many optimization problems the goal state itself is the solution

The state space is a set of *complete* configurations

Search is about finding the **optimal** configuration (as in TSP) or just a **feasible** configuration (as in scheduling problems)

In such cases, one can use **iterative improvement**, or **local search**, methods

An evaluation, or **objective**, function  $h$  must be available that measures the **quality** of each state

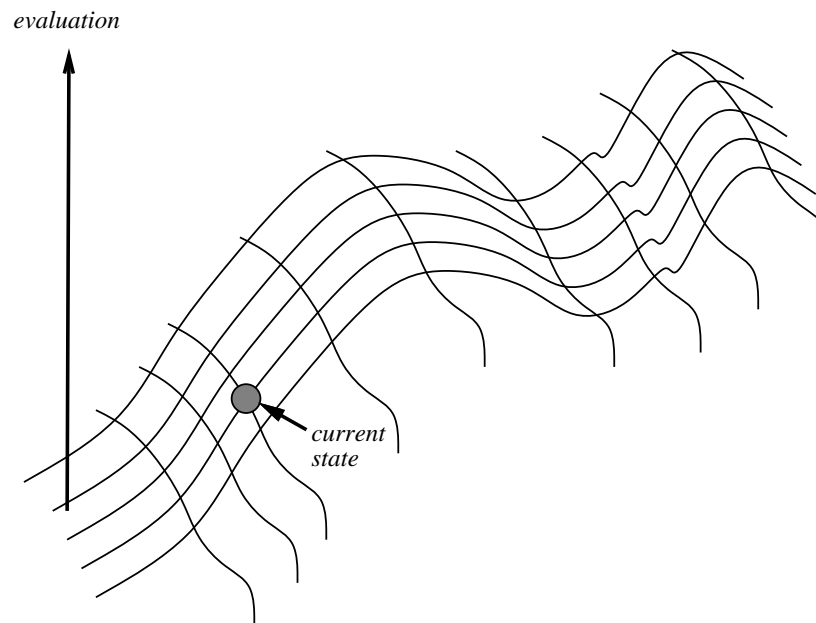
**Main Idea:** Start with a random initial configuration and make small, local changes to it that improve its quality

# Local Search: The Landscape Metaphor

Ideally, the evaluation function  $h$  should be *monotonic*: the closer a state to an optimal goal state the better its  $h$ -value.

Each state can be seen as a point on a surface.

The search consists in moving on the surface, looking for its highest peaks (or, lowest valleys): the optimal solutions.

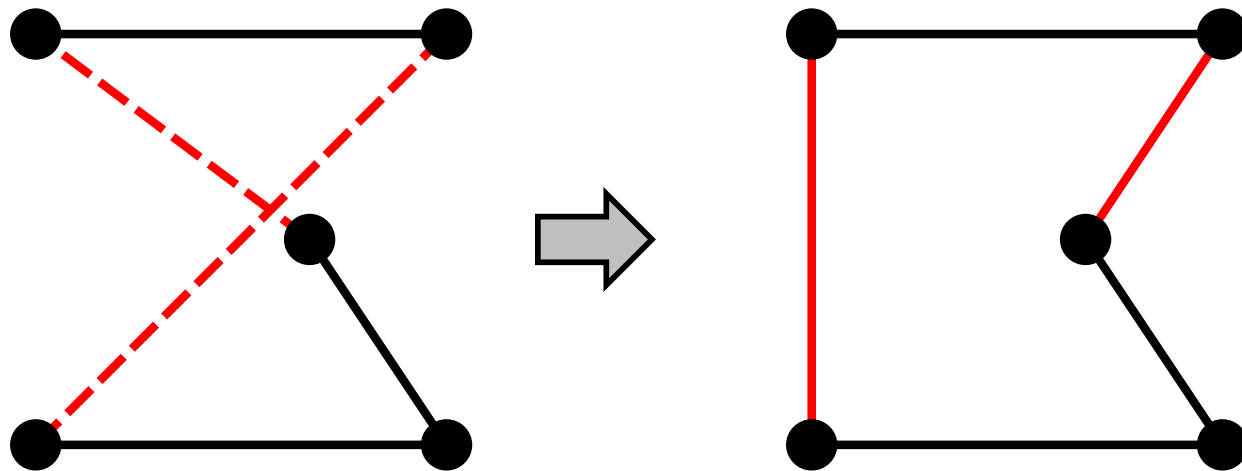


# Local Search Example: TSP

TSP: Travelling Salesperson Problem

$h$  = length of the tour

**Strategy:** Start with any complete tour, perform pairwise exchanges

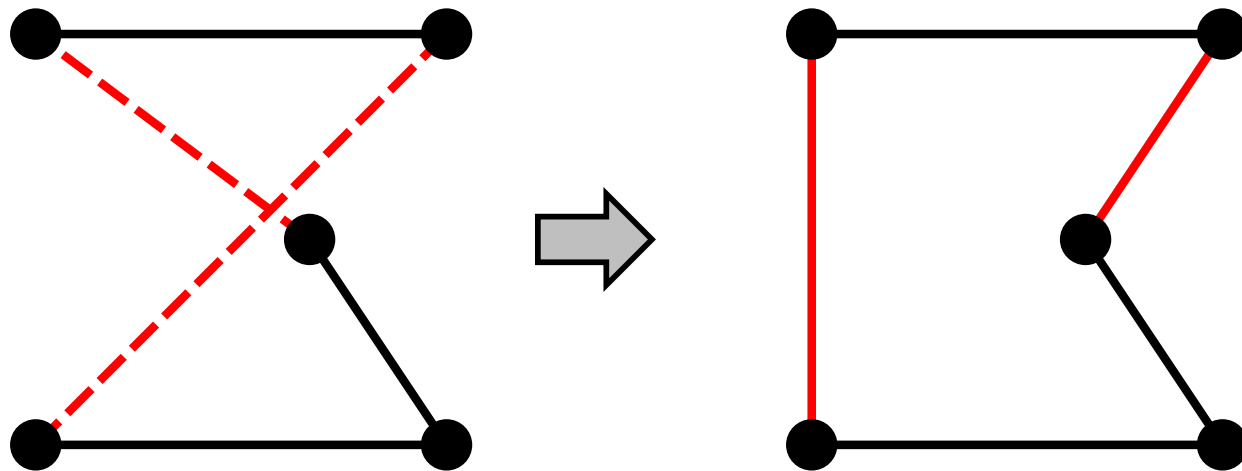


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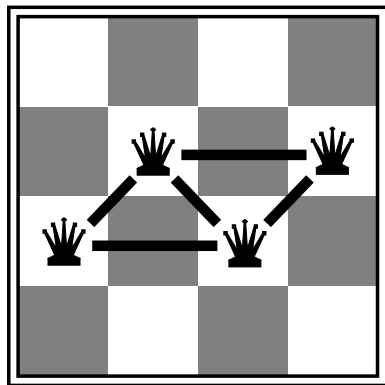
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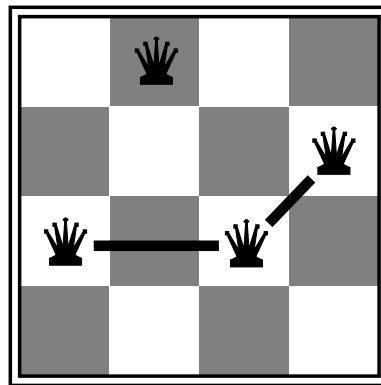
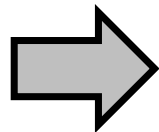
Variants of this approach get within 1% of optimal very quickly with thousands of cities

# Local Search Example: $n$ -queens

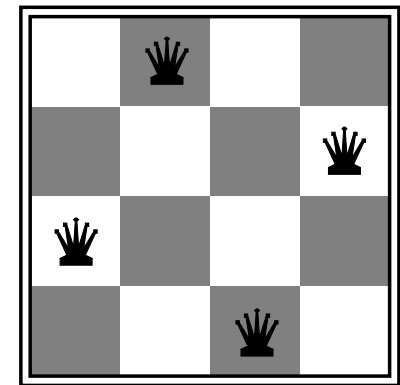
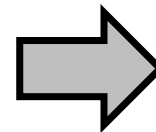
- Put  $n$  queens on an  $n \times n$  board with no two queens on the same row, column, or diagonal
- $h$  = number of conflicts
- **Strategy:** Move a queen to reduce number of conflicts



$h = 5$



$h = 2$

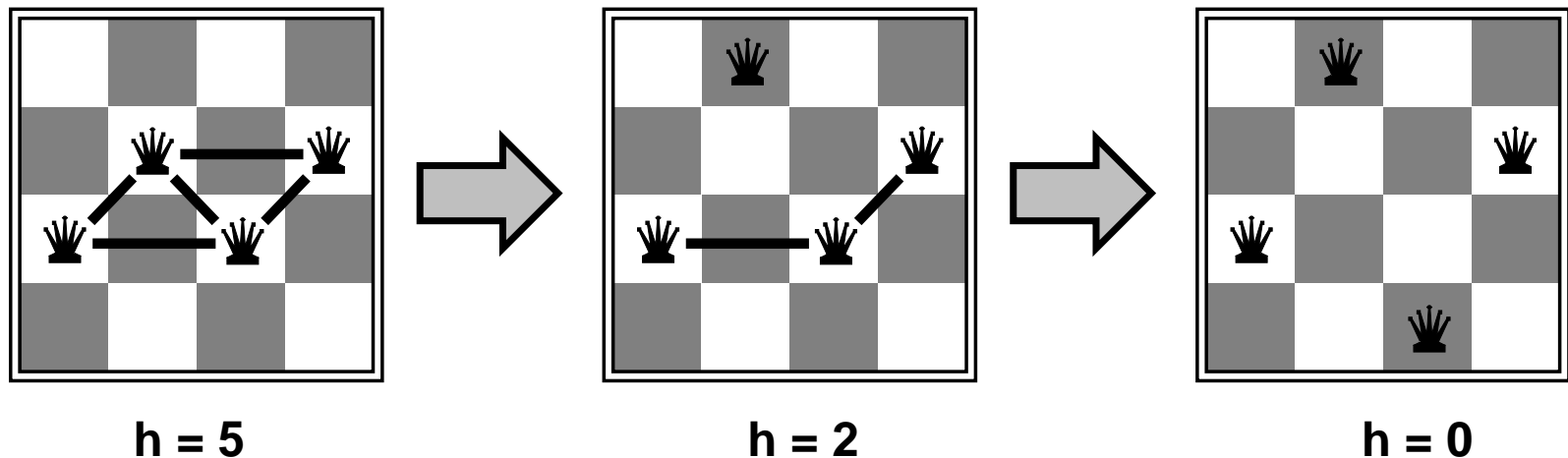


$h = 0$



# Local Search Example: $n$ -queens

- Put  $n$  queens on an  $n \times n$  board with no two queens on the same row, column, or diagonal
- $h$  = number of conflicts
- **Strategy:** Move a queen to reduce number of conflicts



Almost always solves  $n$ -queens problems almost instantaneously for very large  $n$ , e.g.,  $n = 10^6$

# Hill-Climbing Search

Aka: gradient descent/ascent search

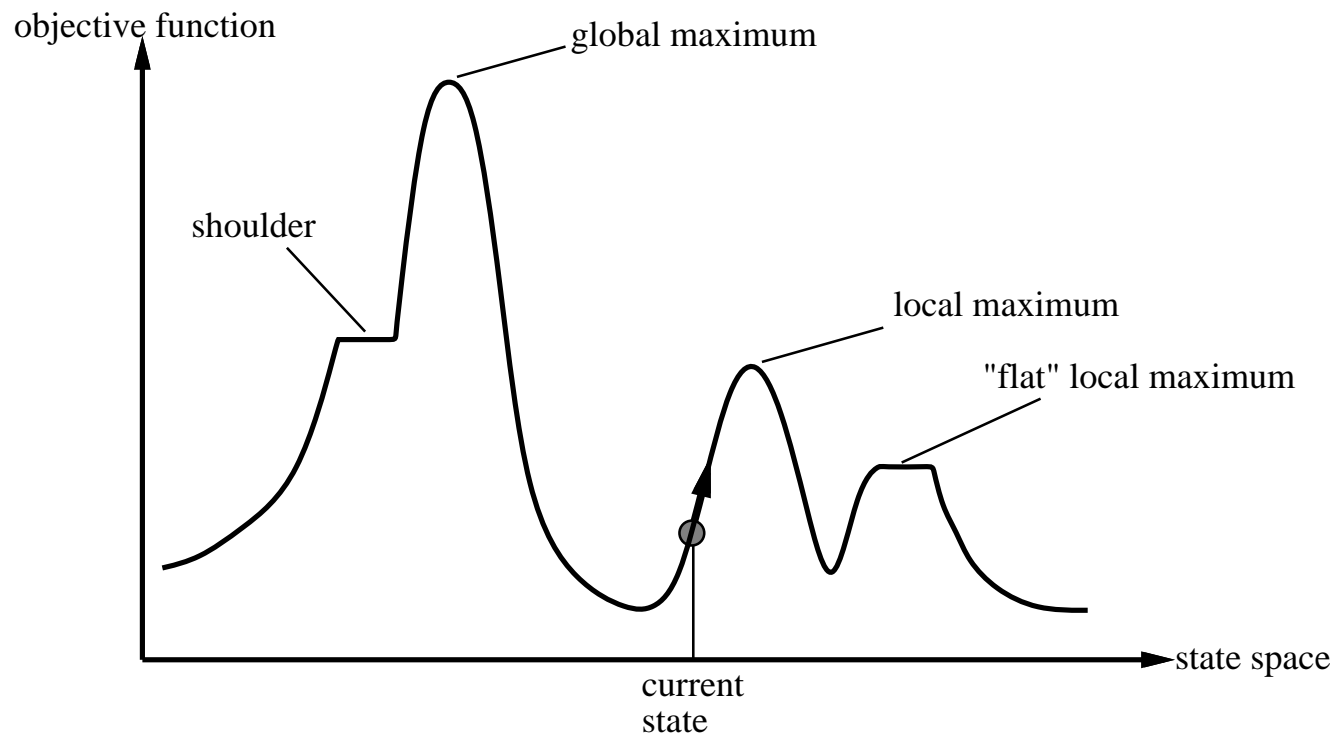
“Like climbing Everest in thick fog with amnesia”

```
function HILL-CLIMBING(problem) returns a state that is a local maximum
  inputs: problem, a problem
  local vars: current, a node
                 neighbor, a node

  current ← MAKE-NODE(INITIAL-STATE[problem])
  loop do
    neighbor ← a highest-valued successor of current
    if VALUE[neighbor] < VALUE[current] then return STATE[current]
    current ← neighbor
  end
```

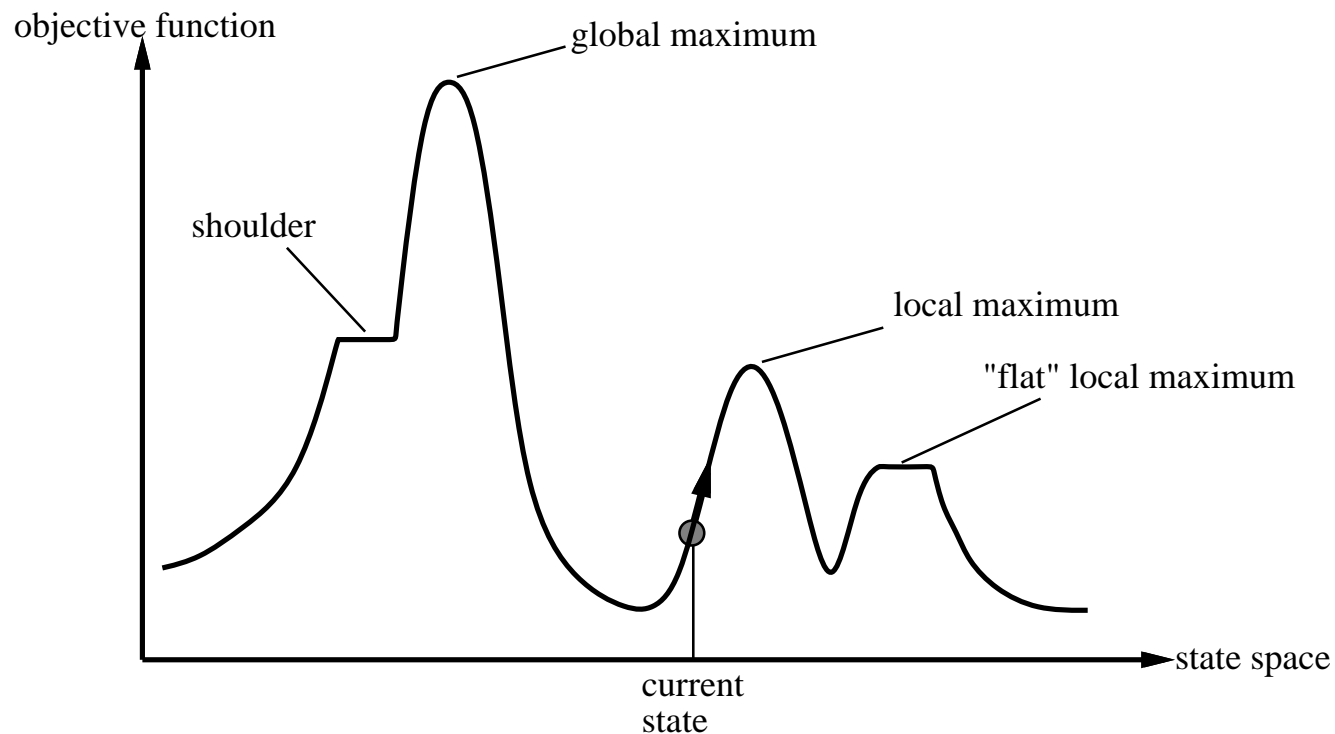
# Hill-Climbing: Shortcomings

- Depending on the initial state, it can get stuck on local maxima
- It may converge very slowly
- In continuous spaces, choosing the step-size is non-obvious



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- Depending on the initial state, it can get stuck on local maxima
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- In continuous spaces, choosing the step-size is non-obvious



However, **true local optima** are surprisingly **rare** in high-dimensional spaces. There often is an escape to a better state

# Hill Climbing: Improvements

Various possible alternatives:

- Restart from a random point in the space
- Look ahead: expand up to  $m > 1$  generations of descendants before choosing best node
- Introduce *noise*: allow down-hill moves sometimes
- Keep  $n > 1$  nodes in the fringe at each step

# Simulated Annealing Search

**Idea:** improve hill-climbing by allowing occasional down-hill steps, to minimize the probability of getting stuck in a local maximum

Down-hill steps taken randomly but with probability that decreases with time

Probability controlled by a given *annealing* schedule for a *temperature* parameter  $T$

If schedule lowers  $T$  slowly enough, search is guaranteed to end in a global maximum

**Catch:** it may take several tries with test problems to devise a good annealing schedule

# Simulated Annealing Algorithm

**function** SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state

**inputs:** *problem*, a problem

*schedule*, a mapping from time to “temperature”

**local vars:** *current*, a node

*next*, a node

*T*, a “temperature” controlling prob. of downward steps

*current* ← MAKE-NODE(INITIAL-STATE[*problem*])

**for** *t* ← 1 **to** ∞ **do**

*T* ← *schedule*[*t*]

**if** *T* = 0 **or** IS-GOAL-STATE(*current*) **then return** *current*

*next* ← a randomly selected successor of *current*

$\Delta E$  ← VALUE[*next*] – VALUE[*current*]

**if**  $\Delta E > 0$  **then** *current* ← *next*

**else** *current* ← *next* only with probability  $\left(\frac{1}{e}\right)^{\frac{|\Delta E|}{T}}$

# Properties of Simulated Annealing

Note:  $\left(\frac{1}{e}\right)^{\frac{|\Delta E|}{T}}$  is directly proportional to  $T$  and  
inversely proportional to  $|\Delta E|$

It can be proven that if  $T$  is decreased slowly enough, the search converges to the best state

This is not necessarily an interesting guarantee

Devised by Metropolis et al., 1953, for physical process modeling

Widely used in VLSI layout, airline scheduling, etc.



# Local Beam Search

**Idea:** improve hill-climbing by keeping  $k$  states instead of 1; choose top  $k$  of all their successors

Not the same as  $k$  searches run in parallel!

Searches that find good states recruit other searches to join them

**Problem:** quite often, all  $k$  states end up on same local maximum

**Solution:** choose  $k$  successors randomly, biased towards good ones

# Genetic Algorithms

Inspired by Darwin's theory of natural selection

Each state is seen as an individual in a population

A **genetic algorithm** applies **selection** and **reproduction** operators to an initial population

The aim is to generate individuals that are most successful, according to a given **fitness function**

It is basically a stochastic local beam search, but with successors generated from **pairs** of states

Local maxima are avoided by giving a nonzero chance of reproduction to low-scoring individuals

# The Basic Genetic Algorithm

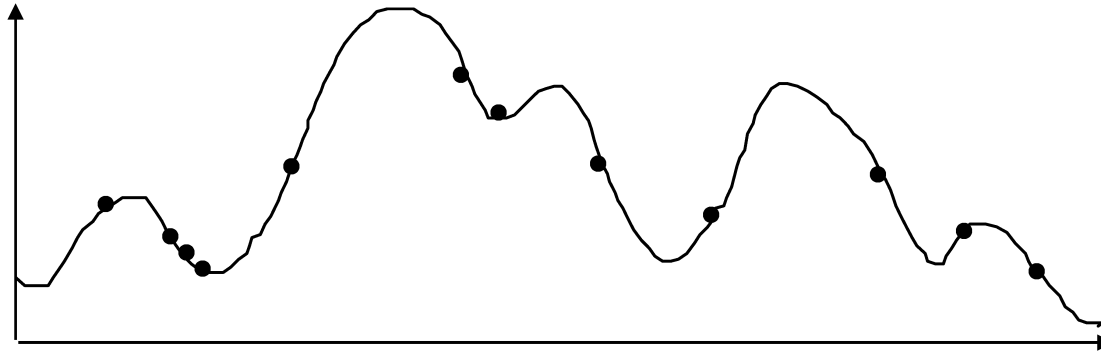
```
function GENETIC-ALGORITHM(population, FITNESS-FN) returns an individual
  inputs: population, a set of individuals
           FITNESS-FN, a function that measures the fitness of an individual

  repeat
    parents ← SELECTION(population, FITNESS-FN)
    population ← REPRODUCTION(parents)
  until some individual is fit enough
  return the best individual in population, according to FITNESS-FN
```

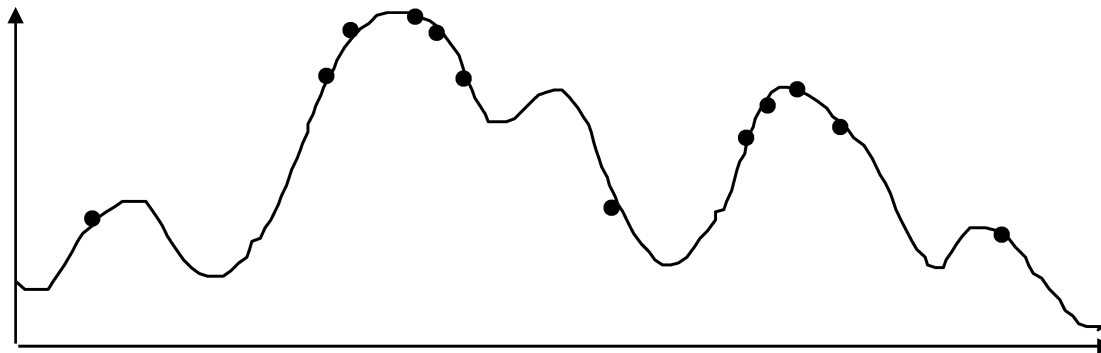
- Variation operators used in REPRODUCTION create the necessary diversity, facilitating novelty
- SELECTION reduces diversity but pushes quality by increasing fitness

# Improving fitness

Abstract example: changes of values of fitness function



Distribution of Individuals in Generation 0



Distribution of Individuals in Generation N

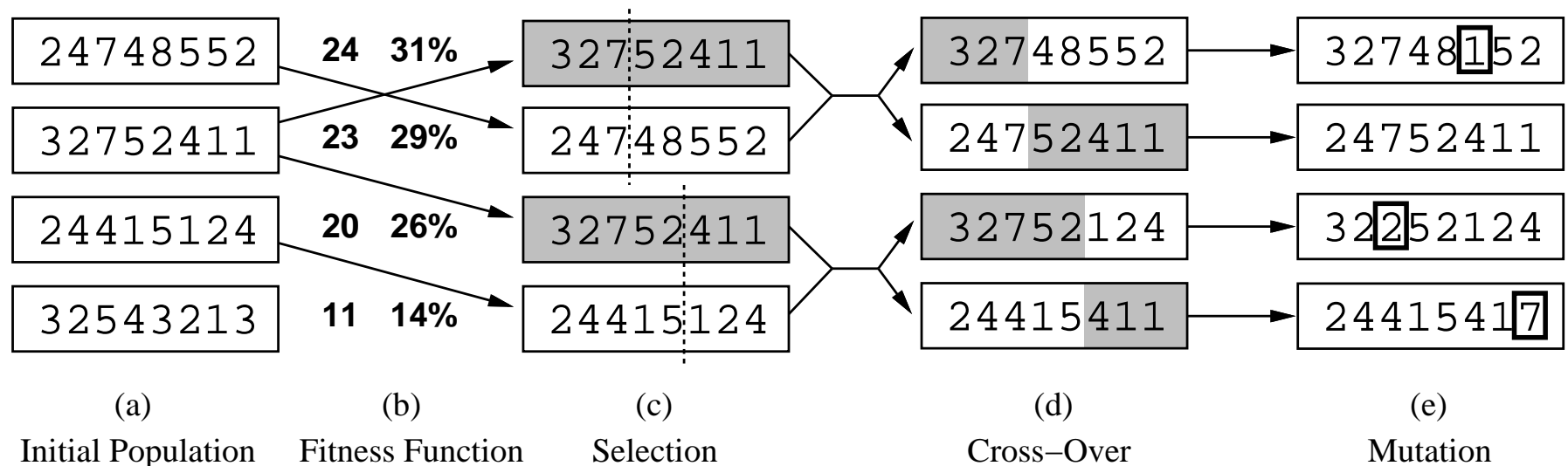
# Genetic Algorithms: Classic Approach

Each state is represented by a finite string over a finite alphabet

Each character in the string is a **gene**

The selection mechanism is **randomized**, with the probability of selection proportional to the fitness measure

Reproduction is accomplished by **crossover** and **mutation**



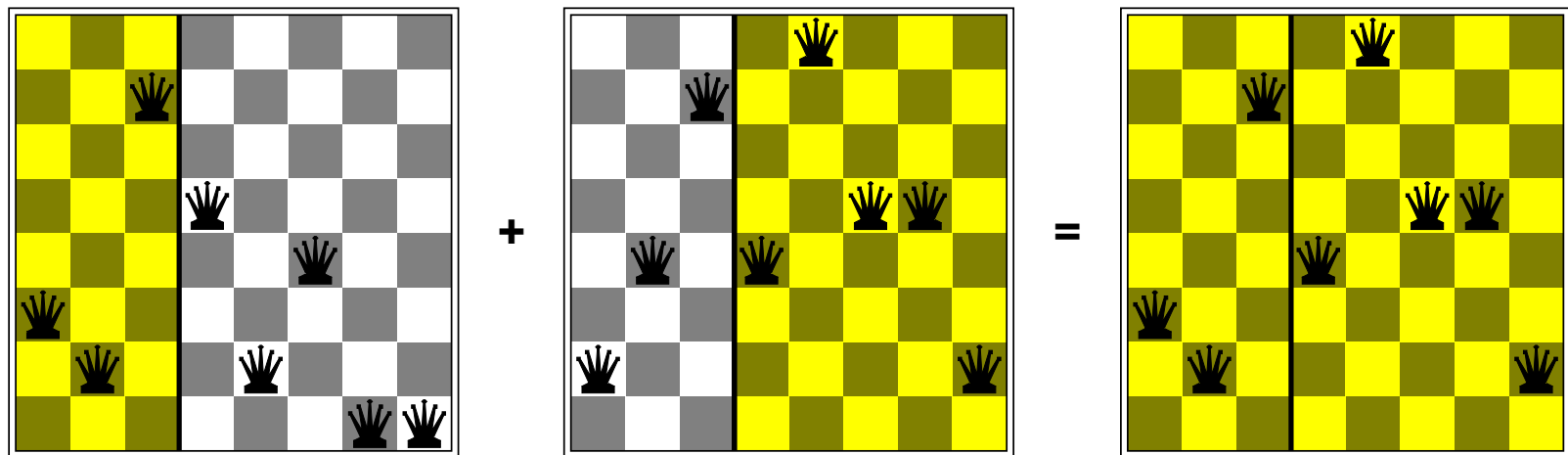
# Possible Encodings

Character strings	0101 ... 1100
Sequences of real numbers	(43.2 -33.1 ... 0.0 89.2)
Tuples of elements	(E11 E3 E7 ... E1 E15)
Lists of rules	(R1 R2 R3 ... R22 R23)
Program elements	(genetic programming)
...	

# Encodings for Genetic Algorithms

Choosing the right encoding of state configurations to strings is crucial.

Crossover helps only if substrings are **meaningful components** that can be reassembled into a new **meaningful configuration**.



**Note:** GAs are not evolution: e.g., real genes encode replication machinery!

# Problem Encoding

**Problem:** Find location that is closest to several given cities

**Population encoding:** Express location  $l$  as a 16-bit string

$$l = 1001010101011100$$

with the first 8 bits representing the location's X-coordinate and the second 8 bits representing the Y-coordinate

**Fitness function:** Median distance of location from each city

**Combination:** Crossover of X-coordinate from one parent and Y-coordinate from the other

**Mutation:** one or more bit flips



# Problem Encoding

**Problem:** Finding the input value that maximizes some function

$$f : [0, 1)^n \rightarrow \mathbb{R}$$

**Population encoding:** Vectors of size  $n$  with elements from  $[0, 1)$

**Combination:** Various options

**Mutation:** randomly replace a value in the vector with one from  $[0, 1)$

# Discrete Recombination

Similar to crossover

Equal probability of receiving each parameter from either parent

Example:

(8, 12, 31, ... , 5) (2, 5, 23, ... , 14)



(2, 12, 31, ... , 14)

# Intermediate Recombination

Each child component is the average of the corresponding parent components

Example:

(8, 12, 31, ... , 5) (2, 5, 23, ... , 14)



(5, 8.5, 27, ... , 9.5)

# GA for the TSP

**Problem:** Find a tour of a given set of cities so that each city is visited only once and total traveled is minimal

**Representation:** An ordered list of city numbers  
(known as order-based GA)

1) London    3) Iowa City    5) Beijing    7) Tokyo  
2) Venice    4) Singapore    6) Phoenix    8) Victoria

Ex.,

CityList1    (3 5 7 2 1 6 4 8)

CityList2    (2 5 7 6 8 1 3 4)

# GA for the TSP

**Combination:** Order 1 crossover (combines inversion and recombination):

1. Copy a randomly selected portion of Parent 1 to Child
2. Fill the blanks in Child with those numbers in Parent 2 from left to right, avoiding duplicates in Child

Parent 1 (3 5 7 2 1 6 4 8)

Parent 2 (2 5 7 6 8 1 3 4)

Child (5 8 7 2 1 6 3 4)

# GA for the TSP

**Mutation:** swap two numbers in the list

Before: (5 8 7 2 1 6 3 4)

After: (5 8 6 2 1 7 3 4)