CS:4350 Logic in Computer Science

Linear Temporal Logic

Cesare Tinelli

Spring 2021

Credits

These slides are largely based on slides originally developed by **Andrei Voronkov** at the University of Manchester. Adapted by permission.

Outline

Linear Temporal Logic
- Computation Tree
- Linear Temporal Logic
- Using Temporal Formulas
- Equivalences of Temporal Formulas
- Expressing Transitions
- Full example

Computation Tree

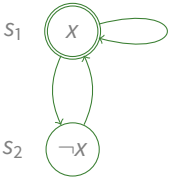
Let $\mathbb{S} = (S, In, T, \mathcal{X}, dom, L)$ be a transition system and $s_0 \in S$ be a state

Computation tree for $\mathbb{S}$ starting at s_0:

Defined as the (possibly infinite) tree C such that

1. every node of C is labeled by a state in S
2. the root of C is labeled by s_0
3. every node in the tree labeled by a state s has a child labeled by a state s' iff $(s, s') \in T$

Computation Trees and Paths



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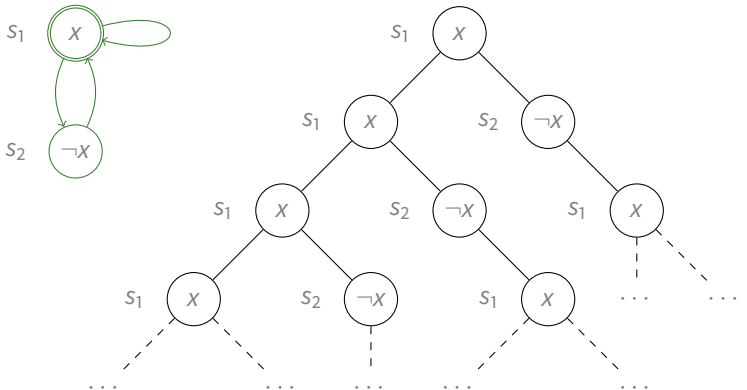

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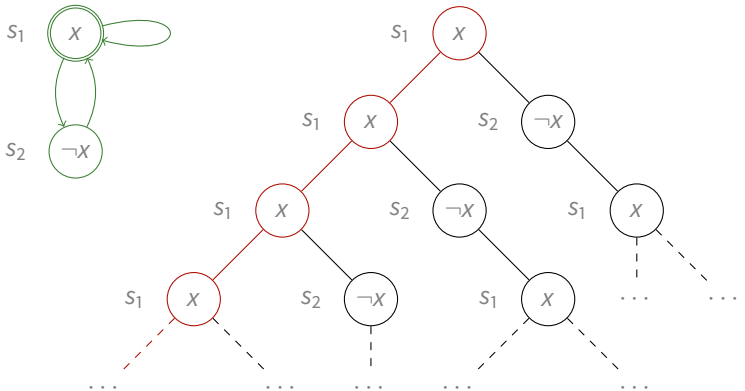

A *computation path* for $\mathbb{S}$ any branch $s_0, s_1, \ldots$ in the tree

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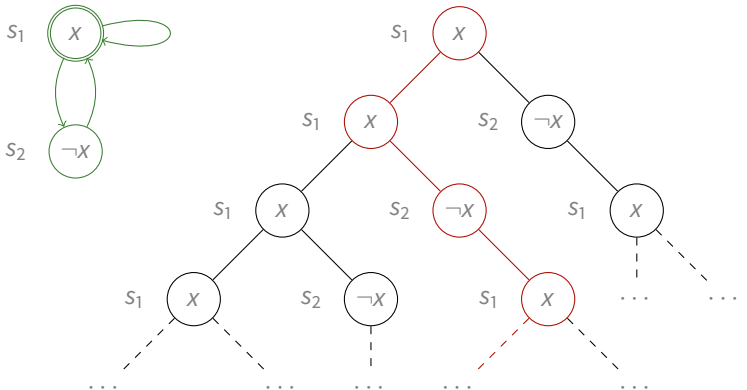

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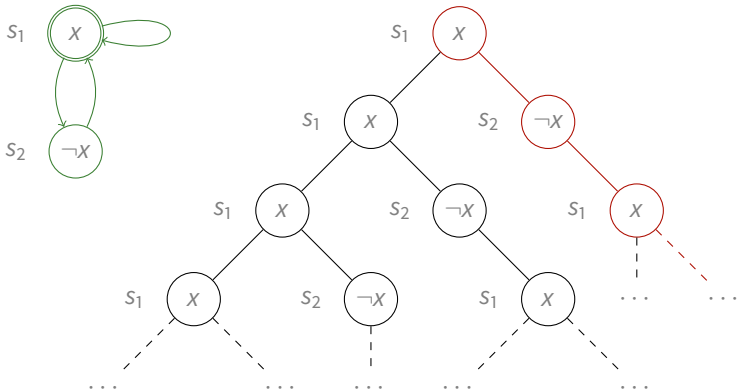

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Computation

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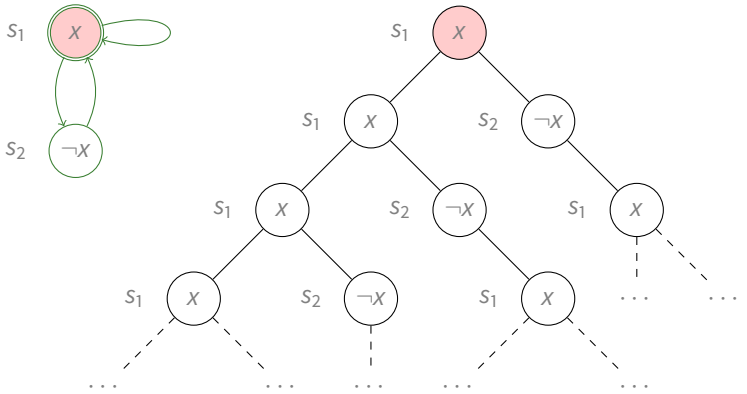

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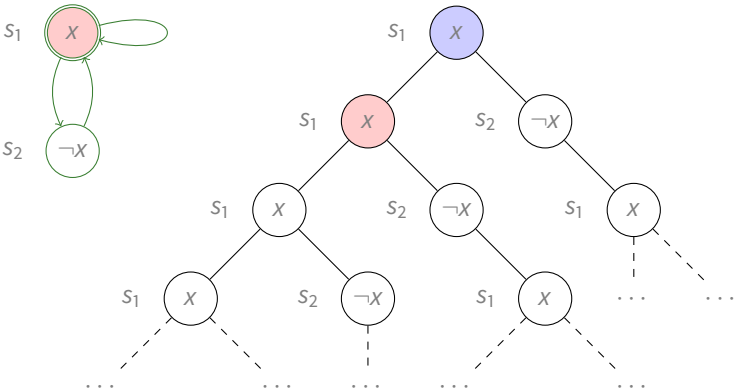

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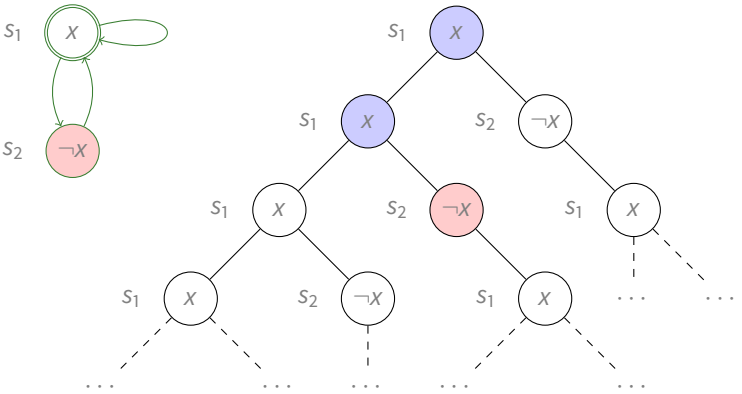

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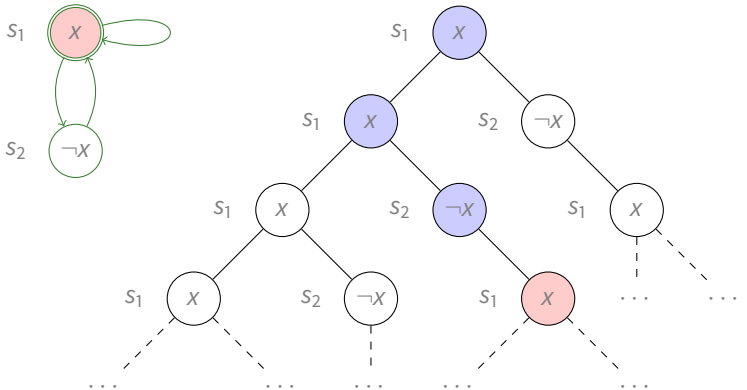

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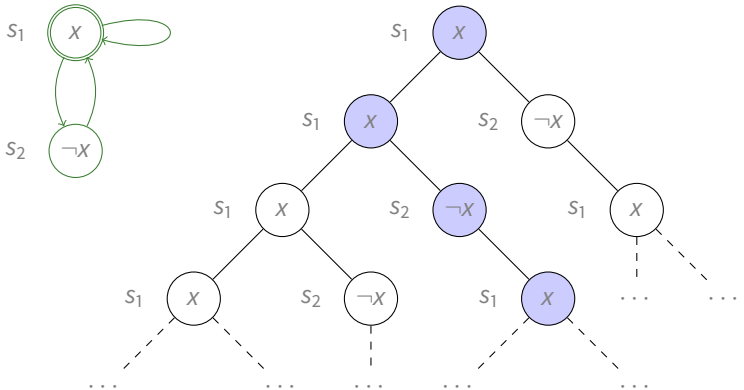

Properties

$$\mathbb{S} = (S, In, T, \mathcal{X}, dom, L)$$

1. The computation paths of $\mathbb{S}$ are exactly the branches in the computation trees for $\mathbb{S}$
2. The subtree of $\mathbb{C}$ rooted at some node s is the computation tree for $\mathbb{S}$ starting at s
   (i.e., every subtree of a computation tree is itself a computation tree)
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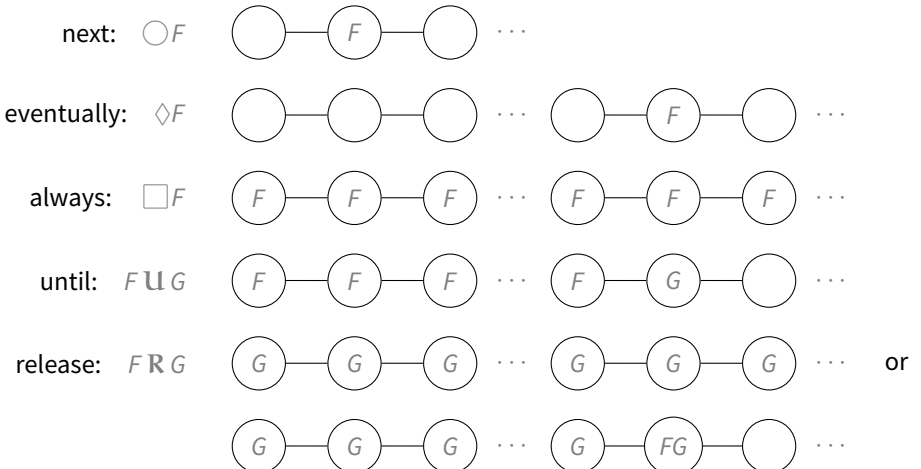
$\bigcirc$	next
$\Box$	always (in the future)
$\Diamond$	sometimes/eventually (in the future)
$\mathcal{U}$	until
$\mathcal{R}$	release

Semantics (intuitive)



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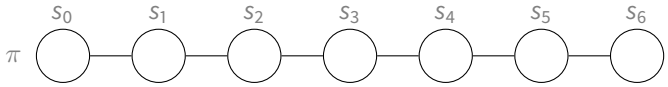
$\pi = s_0, s_1, s_2, \ldots$, sequence of states
$\pi_i = s_i, s_{i+1}, s_{i+2}, \ldots$, subsequence of π starting at $i \geq 0$

F, an LTL formula



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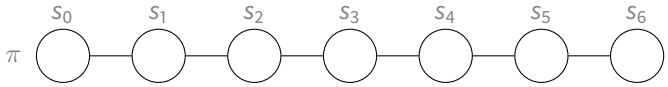

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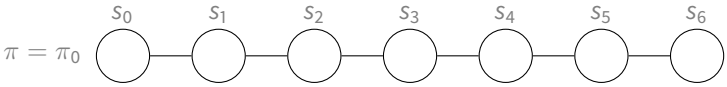

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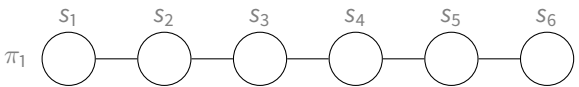

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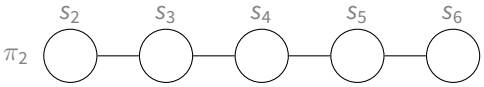

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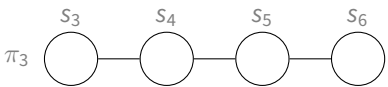

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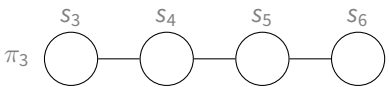

F holds on π or π *satisfies F*, written $\pi \models F$, iff *F holds on* π_0, written $\pi_0 \models F$, where $\pi_i \models F$ is defined for all $i \geq 0$ by induction on F

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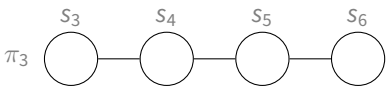

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We will informally say that *F holds in* s_i to mean that F holds on π_i

Semantics, formally

$\pi_i = s_i, s_{i+1}, s_{i+2}, \ldots$

Atomic formulas hold on π_i iff they hold in s_i:

1. $\pi_i \models x = v$ if $s_i \models x = v$

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2. $\pi_i \models \top$ and $\pi_i \not\models \bot$
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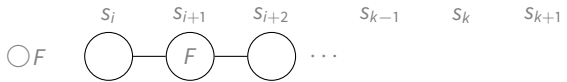

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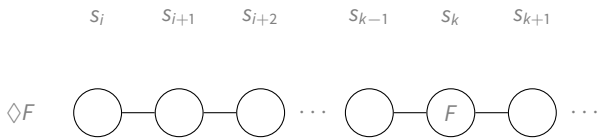

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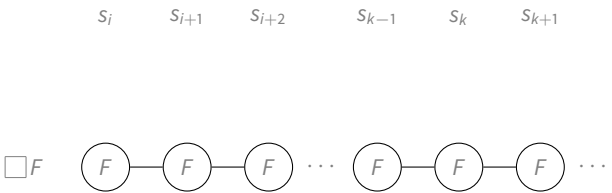

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7. $\pi_i \models F \,\mathfrak{U}\, G$ if for some $k \geq i$ we have $\pi_k \models G$ and $\pi_i \models F, \ldots, \pi_{k-1} \models F$

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   or for some $k \geq i$ and all $j = i, \ldots, k$ we have $\pi_j \models G$ and $\pi_k \models F$

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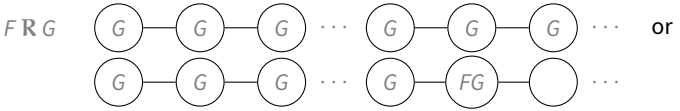

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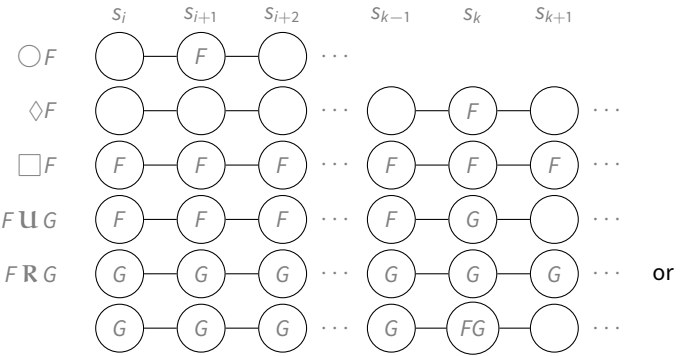

Example

	0	1	2	3	4	5	6	7	8	9	10	11	12	...
p	1	1	1	1	1	1	1	1	0	0	1	1	1	1^ω
q	0	0	0	0	0	0	0	0	1	0	0	1	0	0^ω
$\bigcirc p$	1	1	1	1	1	1	1	0	0	1	1	1	1	1^ω
$\Diamond q$	1	1	1	1	1	1	1	1	1	1	1	1	0	0^ω
$\Box p$	0	0	0	0	0	0	0	0	0	1	1	1	1	1^ω
$p \,\mathbf{U}\, q$	1	1	1	1	1	1	1	1	1	0	1	1	0	0^ω
a	0	0	1	0	0	1	0	0	1	0	1	0	0	0^ω
b	1	1	1	1	1	1	0	1	1	1	1	0	1	1^ω
$a \,\mathbf{R}\, b$	1	1	1	1	1	1	0	1	1	1	1	0	0	0^ω

Notation: v^ω denotes the infinite repetition of v

Standard properties?

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For an LTL formula F we can consider two kinds of properties of $\mathbb{S}$:

1. does F hold on some computation path for $\mathbb{S}$ from an initial of $\mathbb{S}$?
2. does F hold on all computation paths for $\mathbb{S}$ from an initial state of $\mathbb{S}$?

Precedences of Connectives and Temporal Operators

Connective	Precedence
$\neg, \bigcirc, \Diamond, \Box$	5
$\mathcal{U}, \mathcal{R}$	4
$\wedge, \vee$	3
$\rightarrow$	2
$\leftrightarrow$	1

- unary temporal operators have the same precedence as $\neg$
- binary temporal operators have higher precedence than the binary Boolean connectives

Expressing Some Properties

1. F holds initially but not later:
2. F never holds in two consecutive states:
3. If F holds in a state s_i, it also holds in all states after s_i:
4. F holds in at most one state:
5. F holds in at least two states:
6. F happens infinitely often:
7. F holds in each even state and does not hold in each odd state (states are counted from 0):
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Meaning of Some Formulas

- $\Diamond \Box F$

$\Diamond$ (eventually)	$\bigcirc$ (next)
$\Box$ (always)	$\mathcal{U}$ (until)
$\mathcal{R}$ (release)	

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- $\Diamond F \wedge \Box (F \rightarrow \bigcirc F)$
- $\Box \Diamond F$
- $F \wedge \Box (F \leftrightarrow \neg \bigcirc F)$

Expressiveness of LTL

Not all *reasonable* properties are expressible in LTL

Example: p holds in all even states

Equivalences: Unwinding Properties

$\Diamond$ (eventually)	$\bigcirc$ (next)
$\Box$ (always)	$\mathbf{U}$ (until)
$\mathbf{R}$ (release)	

$$\Diamond F \equiv F \vee \bigcirc \Diamond F$$

$$\Box F \equiv F \wedge \bigcirc \Box F$$

$$F \mathbf{U} G \equiv G \vee (F \wedge \bigcirc (F \mathbf{U} G))$$

$$F \mathbf{R} G \equiv G \wedge (F \vee \bigcirc (F \mathbf{R} G))$$

Equivalences: Negation of Temporal Operators

$\Diamond$ (eventually)	$\bigcirc$ (next)
$\Box$ (always)	$\mathcal{U}$ (until)
$\mathcal{R}$ (release)	

$$\neg \bigcirc F \equiv \bigcirc \neg F$$

$$\neg \Diamond F \equiv \Box \neg F$$

$$\neg \Box F \equiv \Diamond \neg F$$

$$\neg (F \,\mathcal{U}\, G) \equiv \neg F \,\mathcal{R}\, \neg G$$

$$\neg (F \,\mathcal{R}\, G) \equiv \neg F \,\mathcal{U}\, \neg G$$

Expressing Temporal Operators Using $\mathcal{U}$

$\Diamond$ (eventually)	$\bigcirc$ (next)
$\Box$ (always)	$\mathcal{U}$ (until)
$\mathcal{R}$ (release)	

$$\Diamond F \equiv \top \,\mathcal{U}\, F$$

$$\Box F \equiv \neg(\top \,\mathcal{U}\, \neg F)$$

$$F \,\mathcal{R}\, G \equiv \neg(\neg F \,\mathcal{U}\, \neg G)$$

Hence, all operators can be expressed using $\bigcirc$ and $\mathcal{U}$

Further Equivalences

$\Diamond$ (eventually)	$\bigcirc$ (next)
$\Box$ (always)	$\mathfrak{U}$ (until)
$\mathfrak{R}$ (release)	

$$\Diamond(F \vee G) \equiv \Diamond F \vee \Diamond G$$

$$\Box(F \wedge G) \equiv \Box F \wedge \Box G$$

But

$$\Box(F \vee G) \not\equiv \Box F \vee \Box G$$

$$\Diamond(F \wedge G) \not\equiv \Diamond F \wedge \Diamond G$$

How to Show that Two Formulas are not Equivalent

Find a path that satisfies one of the formulas but not the other

Example: for $\Box(F \vee G)$ and $\Box F \vee \Box G$

Formalization: Variables and Domains

variable	domain	explanation
st_coffee	$\{ 0, 1 \}$	drink storage contains coffee
st_beer	$\{ 0, 1 \}$	drink storage contains beer
disp	$\{ \mathit{none}, \mathit{beer}, \mathit{coffee} \}$	content of drink dispenser
coins	$\{ 0, 1, 2, 3 \}$	number of coins in the slot
customer	$\{ \mathit{none}, \mathit{student}, \mathit{prof} \}$	customer

Transitions

1. *Recharge* which results in the drink storage having both beer and coffee.
2. *Customer_arrives*, after which a customer appears at the machine.
3. *Customer_leaves*, after which the customer leaves.
4. *Coin_insert*, when the customer inserts a coin in the machine.
5. *Dispense_beer*, when the customer presses the button to get a can of beer.
6. *Dispense_coffee*, when the customer presses the button to get a cup of coffee.
7. *Take_drink*, when the customer removes a drink from the dispenser.

Reasoning About Transitions

Consider the following properties:

1. *one cannot have two beers in a row without inserting a coin*
2. *If we never have two recharge transitions in a row, then the next transition after a recharge must be a customer arrival*

Note that they are about transitions, not states

How can one represent these properties?

Introduce a state variable denoting the next transition

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Example

tr with domain $\{ \mathit{recharge}, \mathit{customer\_arrives}, \mathit{coin\_insert}, \ldots \}$

$$
\begin{aligned}
\mathit{Recharge} &\stackrel{\text{def}}{=} && \mathsf{tr} = \mathit{recharge} \wedge \mathsf{customer} = \mathit{none} \wedge \\
&&& \mathsf{st\_coffee}' \wedge \mathsf{st\_beer}' \wedge \\
&&& \mathit{only}(\mathsf{st\_coffee}, \mathsf{st\_beer}, \mathsf{tr}) \\
\mathit{Customer\_arrives} &\stackrel{\text{def}}{=} && \mathsf{tr} = \mathit{customer\_arrives} \wedge \mathsf{customer} = \mathit{none} \wedge \\
&&& \mathsf{customer}' \neq \mathit{none} \wedge \\
&&& \mathit{only}(\mathsf{customer}, \mathsf{tr}) \\
\mathit{Coin\_insert} &\stackrel{\text{def}}{=} && \mathsf{tr} = \mathit{coin\_insert} \wedge \\
&&& \mathsf{customer} \neq \mathit{none} \wedge \mathsf{coins} \neq 3 \wedge \\
&&& (\mathsf{coins} = 0 \rightarrow \mathsf{coins}' = 1) \wedge \\
&&& (\mathsf{coins} = 1 \rightarrow \mathsf{coins}' = 2) \wedge \\
&&& (\mathsf{coins} = 2 \rightarrow \mathsf{coins}' = 3) \wedge \\
&&& \mathit{only}(\mathsf{coins}, \mathsf{tr})
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$$\Box(\bigwedge_{v \in \mathit{dom}(\mathrm{customer})} (\mathrm{customer} = v \wedge \bigcirc \mathrm{customer} \neq v) \rightarrow \mathrm{tr} = \mathit{customer\_arrives} \vee \mathrm{tr} = \mathit{customer\_leaves})$$

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2. If the system is occasionally recharged, then after each *dispense_beer* the customer will leave:

Representing Temporal Properties of Transitions

1. If somebody inserts a coin twice in a row and then immediately gets a beer, the amount of coins in the slot will not change:

$$\bigwedge_{v \in dom(\mathrm{coin})} \Box(\mathrm{coin} = v \wedge \mathrm{tr} = \mathit{coin\_insert} \wedge \bigcirc \mathrm{tr} = \mathit{coin\_insert} \wedge \bigcirc\bigcirc \mathrm{tr} = \mathit{dispense\_beer} \rightarrow \bigcirc\bigcirc\bigcirc \mathrm{coin} = v)$$

2. If the system is occasionally recharged, then after each *dispense_beer* the customer will leave:

$$\Box\Diamond \mathrm{tr} = \mathit{recharge} \rightarrow \Box(\mathrm{tr} = \mathit{dispense\_beer} \rightarrow \Diamond \mathrm{tr} = \mathit{customer\_leaves})$$

Exercise, Dimmable Lamp

Device A lamp with two buttons that can be

- off
- on but dimmed at medium intensity
- on at full intensity

Actions

1. pushing the first button (set): switches light from off to medium intensity or from medium to full intensity
2. pushing the second button (reset): switches light off
3. doing nothing (none): results just in time passing

Constraints

1. Pushing the first button has no effect if done immediately after a reset
2. Pushing the second button has no effect if done immediately after a set

Exercise, Modeling device as a transition system

State variables

variable	domain	explanation
a	$\{ set, reset, none \}$	actions/transitions
s	$\{ off, on1, on2 \}$	lamp status
st	$\{ 0, 1 \}$	time counter for set
rt	$\{ 0, 1 \}$	time counter for reset

Exercise, Modeling device as a transition system

Initial state formula

$$\mathsf{s} = \mathit{off} \land \mathsf{st} = \mathit{1} \land \mathsf{rt} = \mathit{1}$$

Transition formulas

$$\begin{aligned}
\mathit{Set} \stackrel{\text{def}}{=}\ & \mathsf{a} = \mathit{set} \land \mathsf{rt} \neq \mathit{0} \land \\
& (\mathsf{s} = \mathit{off} \land \mathsf{s}' = \mathit{on1} \lor \mathsf{s} \neq \mathit{off} \land \mathsf{s}' = \mathit{on2}) \land \\
& \mathsf{st}' = \mathit{0} \land \mathit{only}(\mathsf{s}, \mathsf{st}, \mathsf{a}) \\
\mathit{Reset} \stackrel{\text{def}}{=}\ & \mathsf{a} = \mathit{reset} \land \mathsf{st} \neq \mathit{0} \land \\
& \mathsf{s}' = \mathit{off} \land \mathsf{rt}' = \mathit{0} \land \mathit{only}(\mathsf{s}, \mathsf{rt}, \mathsf{a}) \\
\mathit{None} \stackrel{\text{def}}{=}\ & \mathsf{a} = \mathit{none} \land \\
& \mathsf{st}' = \mathit{1} \land \mathsf{rt}' = \mathit{1} \land \mathit{only}(\mathsf{st}, \mathsf{rt}, \mathsf{a})
\end{aligned}$$

Exercise, Temporal properties about the lamp

1. The lamp is initially off.
2. Resetting when the lamp is on turns it off.
3. Resetting always turns the lamp off.
4. Setting when the lamp is off turns it on.
5. Setting when the lamp is half-on turns it fully on.
6. A reset cannot immediately follow a set and vice versa.
7. Setting when the lamp is fully on has no effect on the light.
8. The lamp is initially off and stays off until the first set.
9. Once off, the lamp stays off until the next set.
10. Two consecutive set actions are enough to turn the lamp fully on.
11. If the lamp is on at any point, it must have been turned on some time before.
12. If the lamp is on, it will eventually be off.
13. The lamp will be on repeatedly.
14. At some point the lamp will burn and stay permanently off.
15. If set occurs infinitely often the lamp will be on infinitely often.

Exercise, formalization of properties

1. $s = \mathit{off}$
2. $\Box(a = \mathit{reset} \wedge s \neq \mathit{off} \rightarrow \bigcirc s = \mathit{off})$
3. $\Box(a = \mathit{reset} \rightarrow \bigcirc s = \mathit{off})$
4. $\Box(a = \mathit{set} \wedge s = \mathit{off} \rightarrow \bigcirc s \neq \mathit{off})$
5. $\Box(a = \mathit{set} \wedge s = \mathit{on1} \rightarrow \bigcirc s = \mathit{on2})$
6. $\Box(a = \mathit{set} \rightarrow \bigcirc a \neq \mathit{reset}) \wedge \Box(a = \mathit{reset} \rightarrow \bigcirc a \neq \mathit{set})$
7. $\Box(a = \mathit{set} \wedge s = \mathit{on2} \rightarrow \bigcirc s = \mathit{on2})$
8. $a = \mathit{set}\, \mathbf{R}\, s = \mathit{off}$
9. $\Box(s = \mathit{off} \rightarrow a = \mathit{set}\, \mathbf{R}\, s = \mathit{off})$
10. $\Box(a = \mathit{set} \wedge \bigcirc a = \mathit{set} \rightarrow \bigcirc\bigcirc s = \mathit{on2})$, also
    $\Box(a = \mathit{set} \rightarrow \bigcirc(a = \mathit{set} \rightarrow \bigcirc s = \mathit{on2}))$
11. $\neg(a \neq \mathit{set}\, \mathbf{U}\, s \neq \mathit{off})$
12. $\Box(s \neq \mathit{off} \rightarrow \Diamond s = \mathit{off})$
13. $\Box(\Diamond s \neq \mathit{off})$
14. $\Diamond(\Box s = \mathit{off})$
15. $\Box\Diamond a \neq \mathit{set} \rightarrow \Box\Diamond s \neq \mathit{off}$

Exercise, formalization of properties

1. $\mathsf{s} = \mathit{off}$
2. $\Box(\mathsf{a} = \mathit{reset} \wedge \mathsf{s} \neq \mathit{off} \rightarrow \bigcirc \mathsf{s} = \mathit{off})$
3. $\Box(\mathsf{a} = \mathit{reset} \rightarrow \bigcirc \mathsf{s} = \mathit{off})$
4. $\Box(\mathsf{a} = \mathit{set} \wedge \mathsf{s} = \mathit{off} \rightarrow \bigcirc \mathsf{s} \neq \mathit{off})$
5. $\Box(\mathsf{a} = \mathit{set} \wedge \mathsf{s} = \mathit{on1} \rightarrow \bigcirc \mathsf{s} = \mathit{on2})$
6. $\Box(\mathsf{a} = \mathit{set} \rightarrow \bigcirc \mathsf{a} \neq \mathit{reset}) \wedge \Box(\mathsf{a} = \mathit{reset} \rightarrow \bigcirc \mathsf{a} \neq \mathit{set})$
7. $\Box(\mathsf{a} = \mathit{set} \wedge \mathsf{s} = \mathit{on2} \rightarrow \bigcirc \mathsf{s} = \mathit{on2})$
8. $\mathsf{a} = \mathit{set}\ \mathbf{R}\ \mathsf{s} = \mathit{off}$
9. $\Box(\mathsf{s} = \mathit{off} \rightarrow \mathsf{a} = \mathit{set}\ \mathbf{R}\ \mathsf{s} = \mathit{off})$
10. $\Box(\mathsf{a} = \mathit{set} \wedge \bigcirc \mathsf{a} = \mathit{set} \rightarrow \bigcirc\bigcirc \mathsf{s} = \mathit{on2})$, also $\Box(\mathsf{a} = \mathit{set} \rightarrow \bigcirc(\mathsf{a} = \mathit{set} \rightarrow \bigcirc \mathsf{s} = \mathit{on2}))$
11. $\neg(\mathsf{a} \neq \mathit{set}\ \mathbf{U}\ \mathsf{s} \neq \mathit{off})$
12. $\Box(\mathsf{s} \neq \mathit{off} \rightarrow \Diamond \mathsf{s} = \mathit{off})$
13. $\Box(\Diamond \mathsf{s} \neq \mathit{off})$
14. $\Diamond(\Box \mathsf{s} = \mathit{off})$
15. $\Box\Diamond \mathsf{a} \neq \mathit{set} \rightarrow \Box\Diamond \mathsf{s} \neq \mathit{off}$

Which of these properties are satisfied by every execution path of the transition system?