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Closed FOL formula is either valid or not wrt model $\mathcal{M}$

Consider $\mathcal{M} = (\mathcal{D}, \delta, \mathcal{I})$ to be static part of snapshot, ie state

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Execution of program $\mathtt{p}$ may change state, ie value of $\mathtt{x}$

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Executing $\mathtt{x} = 4;$ results in $\mathcal{M}$ such that $\mathcal{M} \not\models \mathrm{x} \doteq 3$

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Need a logic to capture state before/after program execution

Rigid versus Flexible Symbols

Signature of program logic defined as in FOL, but:

In addition there are program variables, attributes, etc.

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- **Rigid** symbols, same interpretation in **all** execution states

  Needed, for example, to hold initial value of program variable

  Logical variables and **built-in functions/predicates** are rigid

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Rigid versus Flexible

- **Rigid symbols, same interpretation in all execution states**

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  Logical variables and built-in functions/predicates are rigid

- **Non-rigid (or flexible) symbols, interpretation depends on state**

  Needed to capture state change after program execution

  Functions modeling program variables and attributes are flexible

Signature of Dynamic Logic (Simple Version)

Given type hierarchy $\mathcal{T}_q = \{\texttt{int}, \texttt{boolean}, \top\}$

Signature $\Sigma = ($**VSym**, **PSym**, **FSym**, **PVSym**, $\alpha)$

Variable Symbols	**VSym** $= \{x_i \mid i \in I\!N\}$
Rigid Predicate Symbols	**PSym**$_r = \{>, >=, \ldots,\}$
Rigid Function Symbols	**FSym**$_r = \{+, -, *, \mathtt{0}, \mathtt{1}, \mathtt{TRUE}, \mathtt{FALSE}\}$
Non-rigid Function Symbols	**FSym**$_{nr} = \{i, j, k, \ldots, p, q, r, \ldots\}$

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Typing function α **for all symbols:**

- $\alpha(\texttt{j}) \in \{\texttt{int}, \texttt{boolean}\}$ **for all** $\texttt{j} \in$ **FSym**$_{nr}$

  When $b : \rightarrow \texttt{boolean}$, **write** $\texttt{boolean b}$, **etc.;, use as program variable**

- **Standard typing for rigid function/predicate symbols**

  For example, $\texttt{TRUE} : \rightarrow \texttt{boolean}$, $\quad > : \texttt{int}, \texttt{int}$

Terms

First-order terms may contain rigid and non-rigid symbols

Different syntactic categories: $\mathsf{FSym}_r \cap \mathsf{FSym}_{nr} = \emptyset$

Program variables are non-rigid (=flexible) constants

Emphasize distinction to variables VSym: call them logical variables

A term containing at least one flexible symbol is flexible, otherwise rigid

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Examples

VSym $= \{x : \mathtt{int}, b : \mathtt{boolean}\}$
FSym$_{nr}$ $= \{\mathtt{int}\ \mathtt{j}, \mathtt{boolean}\ \mathtt{p}\}$

Well-formed terms: $\mathtt{j}+x$, $\mathtt{j}$, b
Ill-formed terms: $\mathtt{j}+b$, $\mathtt{j}+\mathtt{p}$

Atomic Programs

Atomic Programs Π_0

- **Assignments** $\mathtt{j} = t$ **with:**
  $\mathtt{z}\ \mathtt{j} \in$ **FSym**$_{nr}$**,** $\quad t$ **term of type** $\mathtt{z}$ **without logical variables**

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Examples

VSym $= \{x : \mathtt{int}, b : \mathtt{boolean}\}$
FSym$_{nr}$ $= \{\mathtt{int}\ \mathtt{j}, \mathtt{boolean}\ \mathtt{p}\}$

Well-formed atomic programs: $\mathtt{j} = \mathtt{j} + 1$**,** $\mathtt{j} = 0$**,** $\mathtt{p} = \mathtt{FALSE}$

Ill-formed atomic programs: $\mathtt{j} = \mathtt{j} + x$**,** $x = 1$**,** $\mathtt{j} \doteq \mathtt{j}$**,** $\mathtt{p} = 0$

Dynamic Logic (Simple Version) Programs

Programs Π

- If π is an atomic program, then $\pi;$ is a program
- If p and q are programs, then pq is a program
- If b is a variable-free term of type `boolean`, p and q programs, then

  if (b) {p} else {q};

  is a program
- If b is a variable-free term of type `boolean`, p a program, then

  while (b) {p};

  is a program

Programs contain no logical variables

Dynamic Logic Syntax Example

Given signature

PSym$_r = \{<\}$

FSym$_r = \{0, +, -\}$

FSym$_{nr} = \{\mathtt{int}\ \mathtt{i}, \mathtt{int}\ \mathtt{r}, \mathtt{int}\ \mathtt{n}\}$

An admissible DL program $\mathtt{p}$**:**

```
i=0;
r=0;
while (i<n) {
  i=i+1;
  r=r+i;
};
r=r+r-n;
```

What does $\mathtt{p}$ **compute?**

Dynamic Logic (Simple Version) Formulas

Dynamic Logic Formulas (DL Formulas)

- **Each FOL formula is a DL formula**

  DL formulas closed under FOL operators and connectives

- **If p is a program and ϕ a DL formula then**

  $\langle \mathrm{p} \rangle \phi$ **is a DL formula**

  $[\mathrm{p}]\,\phi$ **is a DL-Formula**

Program variables are constants: never bound in quantifiers

Programs contain no logical variables

The operators $\langle\ \rangle$ and $[\ ]$ can be arbitrarily nested

Dynamic Logic Syntax Example

Check for syntactic well-formedness and derive the signature

$\forall y. ((\langle \mathtt{x} = 1; \rangle \mathtt{x} \doteq y)$ **<->** $(\langle \mathtt{x} = 1 * 1; \rangle \mathtt{x} \doteq y))$ **Syntax ?**

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$\exists \mathtt{x}. ([\mathtt{x} = \mathtt{1};] (\mathtt{x} \doteq 1))$ **Syntax ?**

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- x **cannot be logical variable, because it occurs in program**
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$\langle \mathtt{x} = 1; \rangle ([\mathbf{while}\ (\mathbf{true})\ \mathbf{\{\}}]\ \mathbf{false})$ **Syntax ?**

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$\forall y. (((\langle \mathtt{x} = 1; \rangle \mathtt{x} \doteq y)$ **<->** $(\langle \mathtt{x} = 1 * 1; \rangle \mathtt{x} \doteq y))$ **ok** **(**$y : \mathtt{int}$**)**

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- x **cannot be program variable, because it is quantified**

$\langle \mathtt{x} = 1; \rangle ([\textbf{while}\ (\textbf{true})\ \textbf{\{\}}]\ \textbf{false})$ **ok** **(**$\mathtt{int\ x}$**)**

- **Program formulas can appear nested**

More Examples of DL Formulas

1. $\mathtt{x} \doteq i$ **&** $\mathtt{y} \doteq j$ **->** $\langle \mathtt{z} = \mathtt{x}; \mathtt{x} = \mathtt{y}; \mathtt{y} = \mathtt{x}; \rangle \mathtt{x} \doteq j$ **&** $\mathtt{y} \doteq i$

2. $\mathtt{x} \doteq 3$ **|** $\mathtt{y} \doteq -2$ **->** $\langle \mathtt{y} = \mathtt{x} * \mathtt{x} - \mathtt{x} + 6; \rangle \mathtt{y} \doteq 0$

3. $\langle$**if** 0 **<=** $\mathtt{a}$ **then {} else {**$\mathtt{a} = -\mathtt{a};$ **}**$\rangle 0$ **<=** $\mathtt{a}$

4. $\langle$**while** $(\mathtt{c}$ **<=** $\mathtt{n} - 1)$ **{**$\mathtt{p} = \mathtt{p} + \mathtt{m}; \mathtt{c} = \mathtt{c} + 1;$ **}**$\rangle$ $\mathtt{p} \doteq \mathtt{m} * \mathtt{m}$

Dynamic Logic Semantics: States

First-order model can be considered as (execution) state

Interpretation of non-rigid symbols can vary from state to state

(eg, program variables)

Interpretation of rigid symbols is the same in all states

(eg, built-in functions and predicates)

State = First-order model:

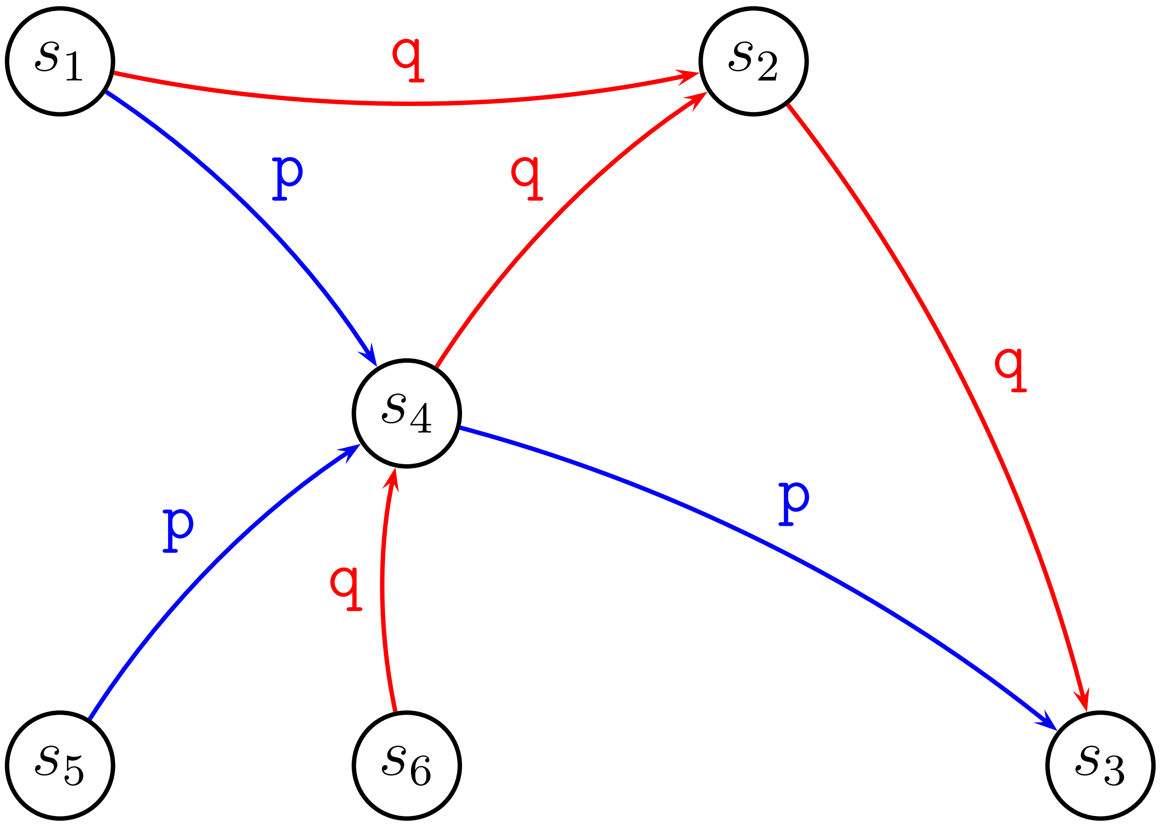
$\mathcal{M} = s = (\mathcal{D}, \delta, \mathcal{I})$ over FSym $=$ FSym$_r \cup$ FSym$_{nr}$

Set of all states s is S

Dynamic Logic Semantics: Kripke Structure

Kripke structure $K = (S, \rho)$

State (model) $s = (\mathcal{D}, \delta, \mathcal{I}) \in S$ **and** $\rho : \Pi \to (S \to S)$ $\quad \rho(\mathtt{p}),\ \rho(\mathtt{q})$

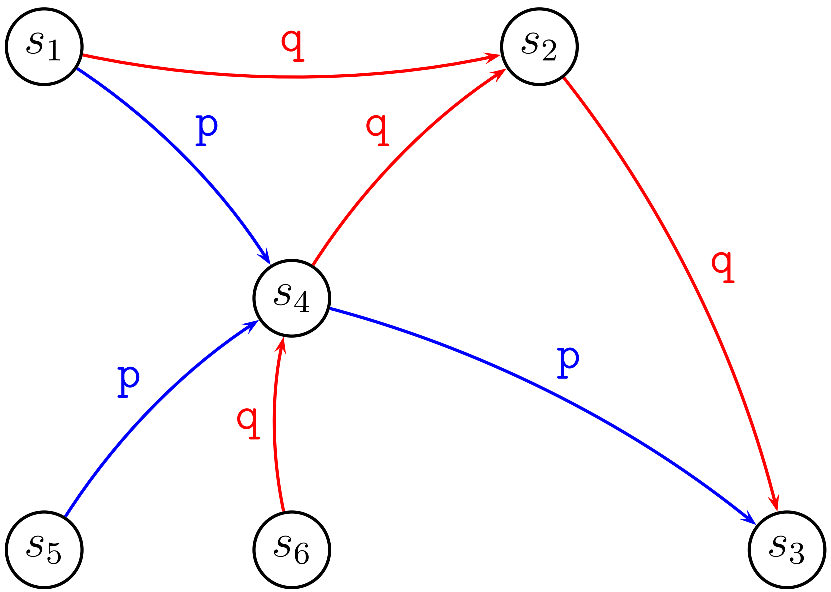


Each state is first-order model $s = (\mathcal{D}, \delta, \mathcal{I})$ **over same domain** $\mathcal{D}$

Dynamic Logic Semantics: Program Formulas

- $s, \beta \models \langle \mathtt{p} \rangle \phi$ **iff** $\rho(\mathtt{p})(s), \beta \models \phi$ **and** $\rho(\mathtt{p})(s)$ **defined**

  $\mathtt{p}$ ***terminates and*** ϕ ***is true in the final state after execution***



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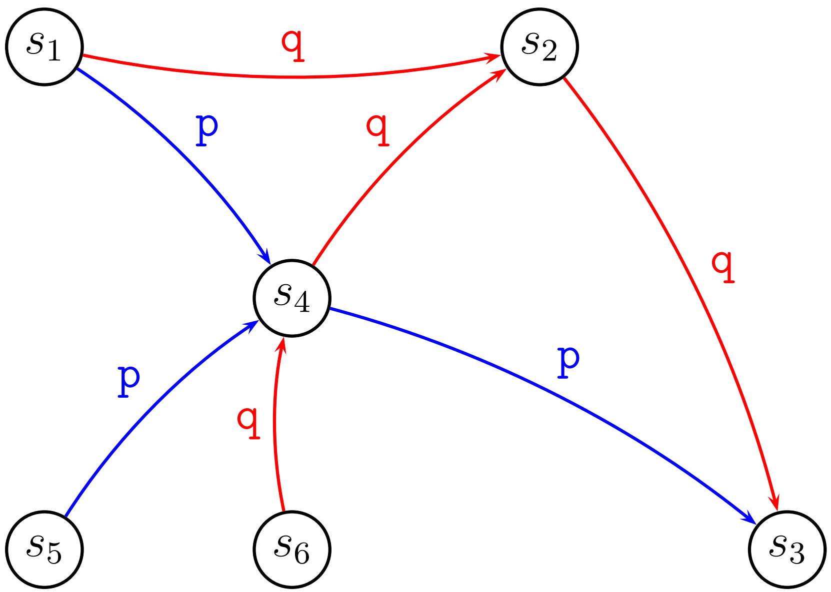
- $s, \beta \models [\mathtt{p}]\, \phi$ **iff** $\rho(\mathtt{p})(s), \beta \models \phi$ **whenever $\rho(\mathtt{p})(s)$ defined**

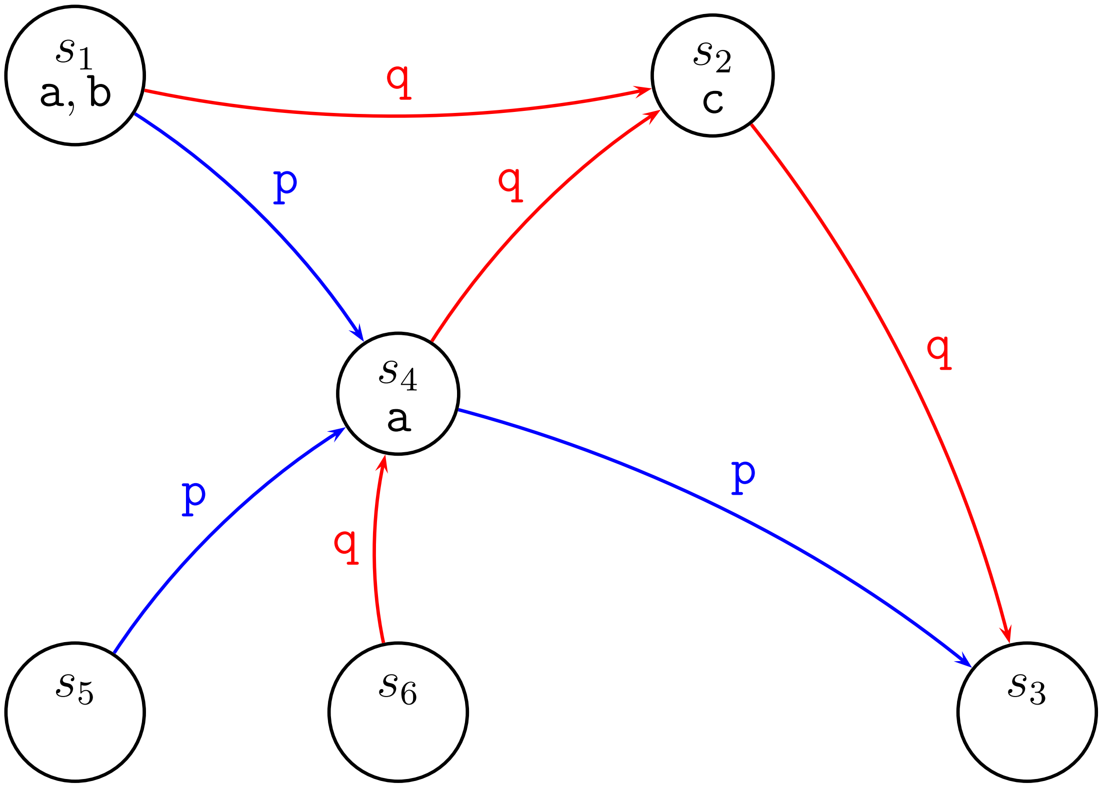
  If $\mathtt{p}$ ***terminates then ϕ is true in the final state after execution***



Dynamic Logic Semantics Example

Boolean program variables

$\textbf{FSym}_{nr} = \{\texttt{boolean a}, \texttt{boolean b}, \texttt{boolean c}, \ldots\}$



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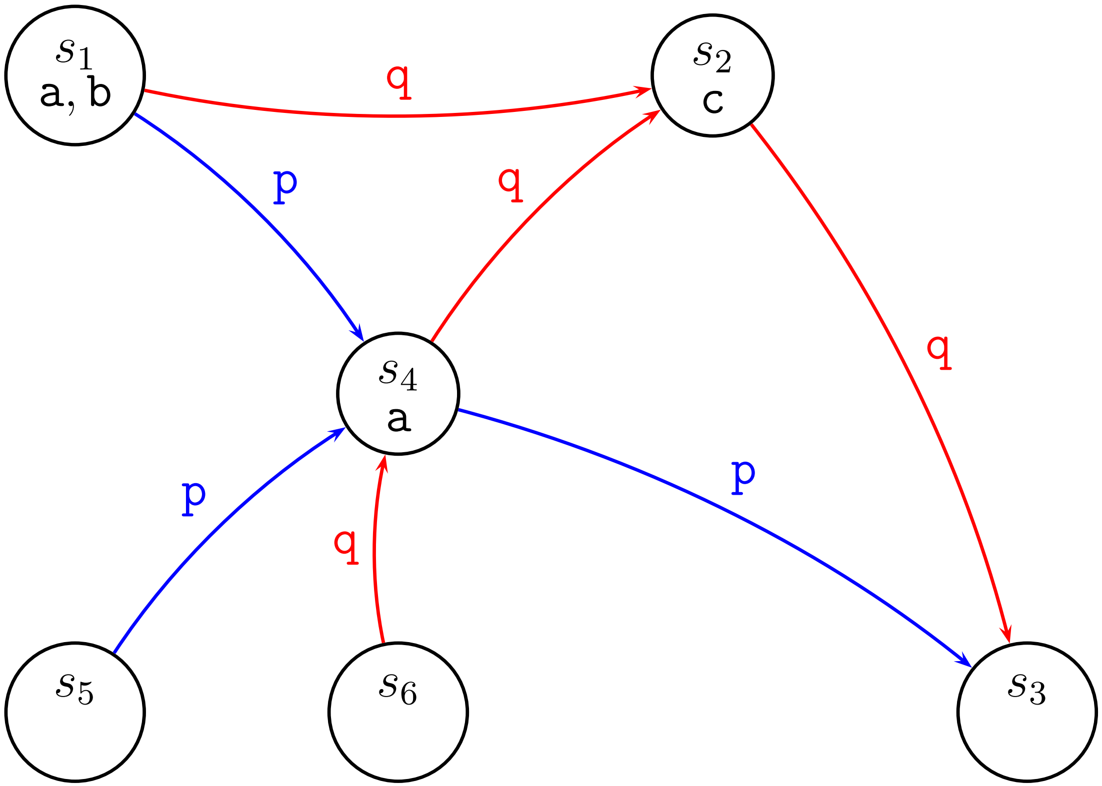

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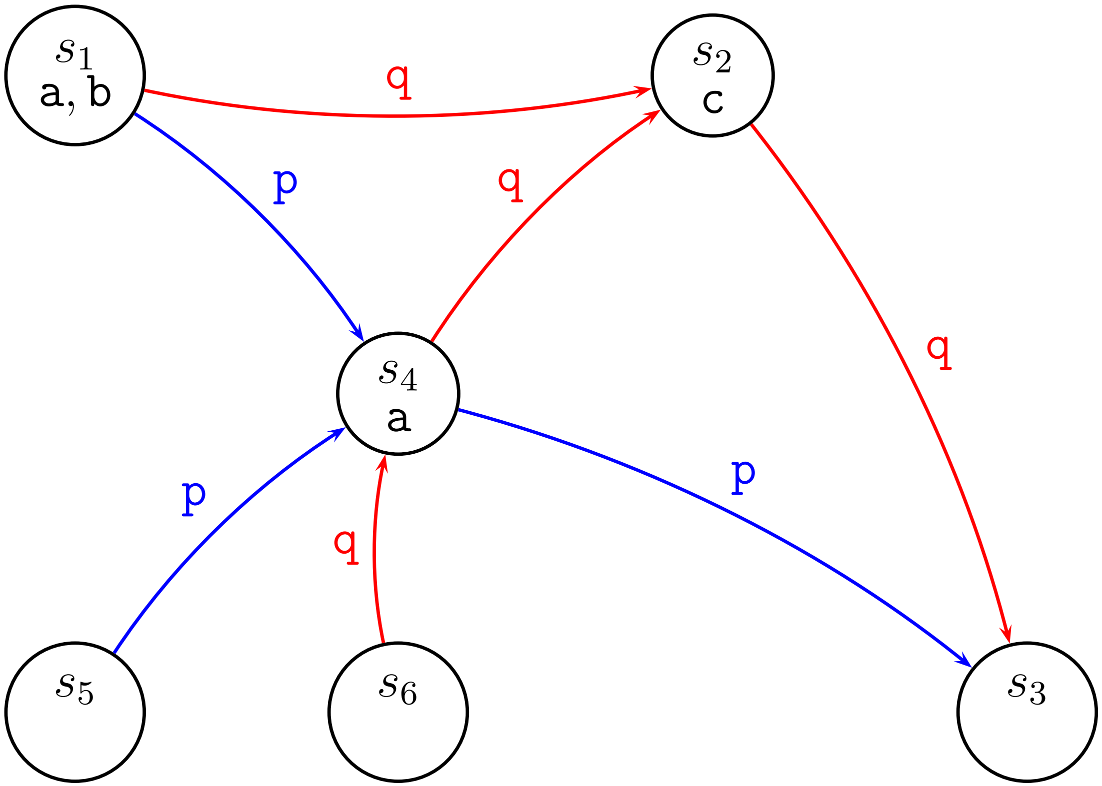

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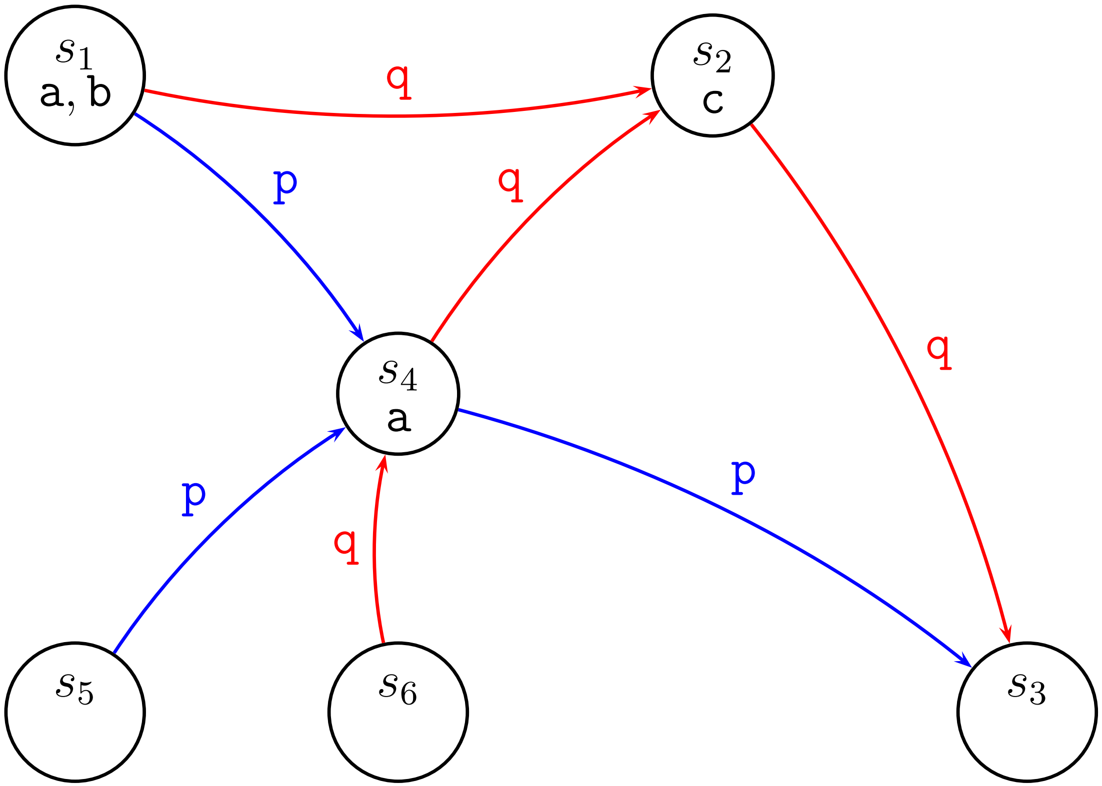

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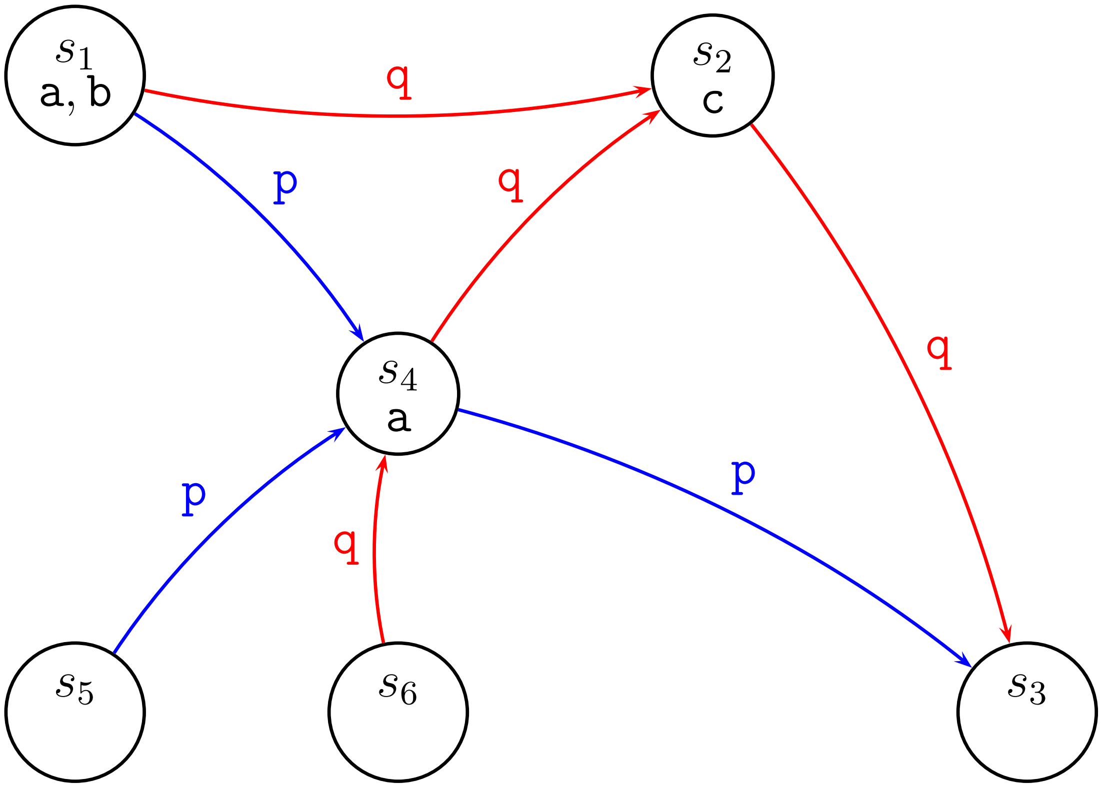

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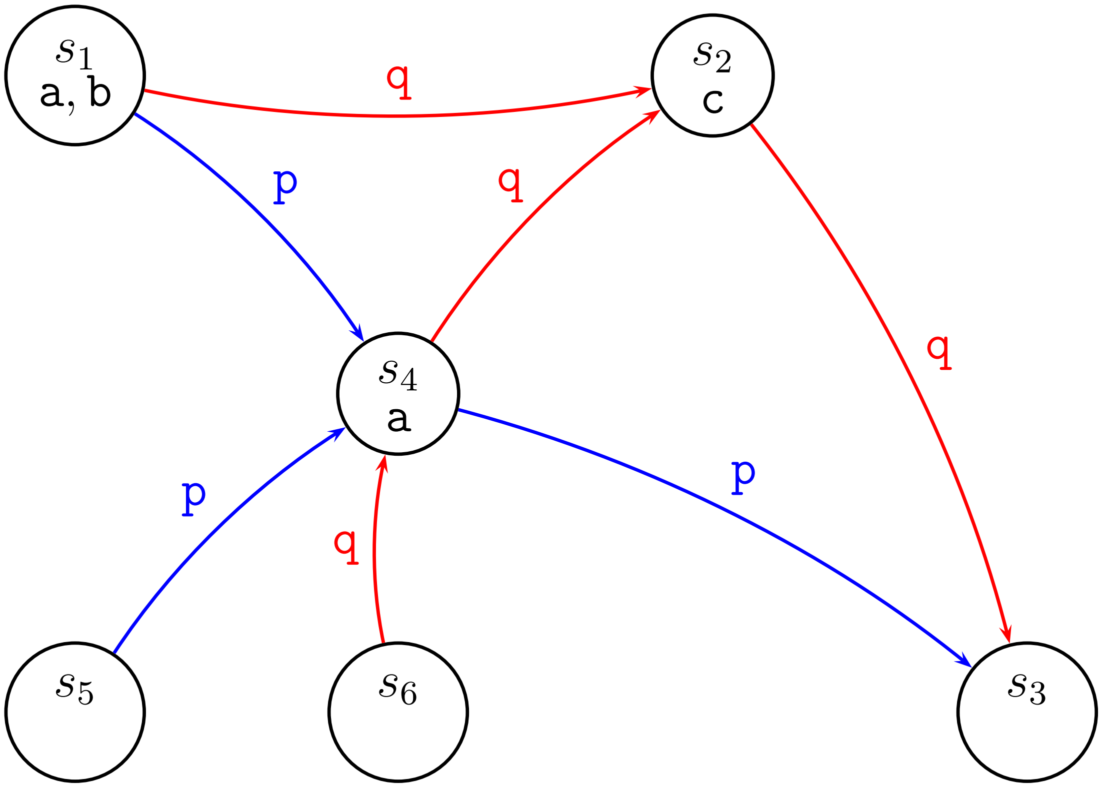

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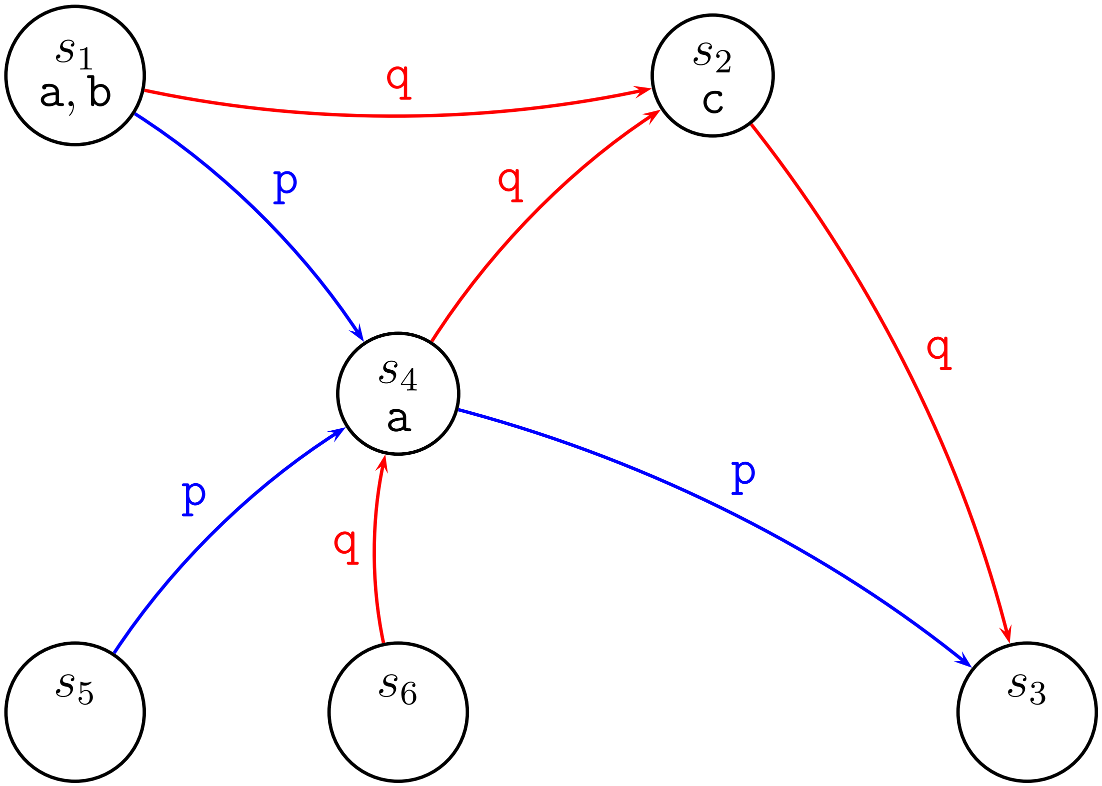

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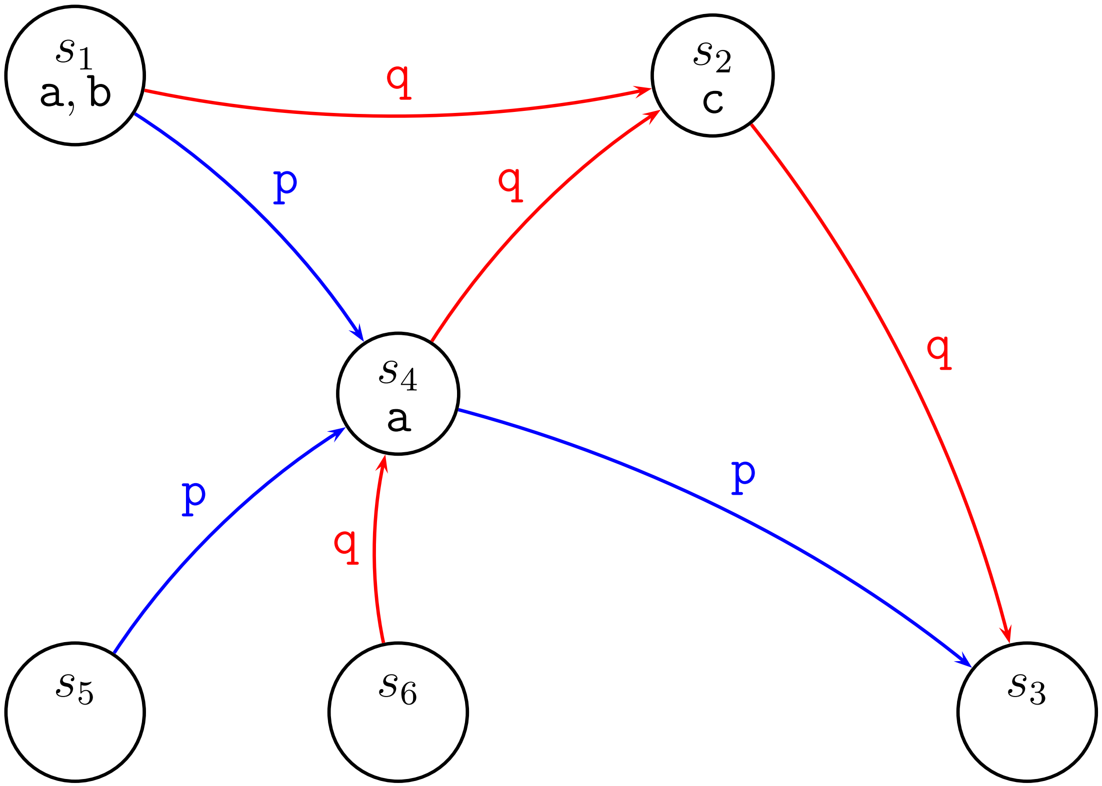

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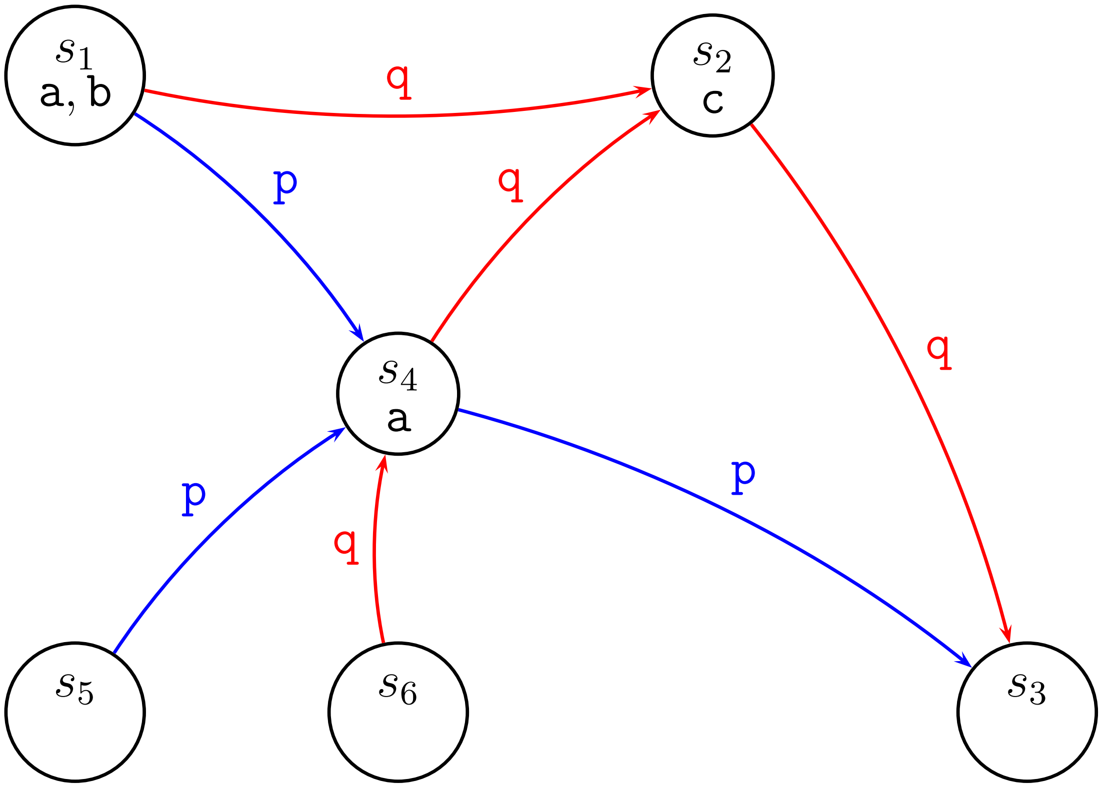

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Program Correctness

- $s, \beta \models \langle \mathtt{p} \rangle \phi$

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- **Duality** $\langle \mathtt{p} \rangle \phi$ **iff** **!** $[\mathtt{p}]$ **!**ϕ

  Exercise: justify this with semantic definitions

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- **Implication** **if** $\langle \mathtt{p} \rangle \phi$ **then** $[\mathtt{p}]\, \phi$

  Total correctness implies partial correctness
  (holds only for deterministic programs)

Semantics of Sequents

Let $\Gamma = \{\phi_1, \ldots, \phi_n\} \subseteq$ **For and** $\Delta = \{\psi_1, \ldots, \psi_m\} \subseteq$ **For**

Recall: $s \models (\Gamma$ **==>** $\Delta)$ **iff** $s \models (\phi_1$**&**$\cdots$**&**$\phi_n)$ **->** $(\psi_1$**|**$\cdots$**|**$\psi_m)$

Semantics of DL sequents should be defined identically with semantics of FOL sequents (assume Γ, Δ are sets of closed DL formulas):

Γ **==>** Δ **is valid** **iff** $s \models (\Gamma$ **==>** $\Delta)$ **in all states** s

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Consequence for program variables

In valid formulas they represent any possible value of their type

Initial States

How to restrict validity to set of initial states $S_0 \subseteq S$?

1. **Design closed FOL formula Init with**

   $s \models \mathbf{Init}$ **iff** $s \in S_0$

2. **Use sequent** $\Gamma, \mathbf{Init}$ **==>** Δ

Later: simple method for specifying initial value of program variables

Dynamic Logic Semantics: States, Updates

- **States $s = (\mathcal{D}, \delta, \mathcal{I})$ all have**
  - **the same domain $\mathcal{D}$ (all objects present from start)**
  - **the same typing function δ (dynamic type never changes)**

  May assume $\rho(\mathtt{p})$ works on interpretations $\mathcal{I}$

  Define $\mathcal{I}, \beta \models \phi$ as $s, \beta \models \phi$, where $s = (\mathcal{D}, \delta, \mathcal{I})$

- **Program variables $\mathtt{j}$ as flexible constants in s with value $\mathcal{I}(\mathtt{j})$**

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- Program variables $\mathtt{j}$ as flexible constants in s with value $\mathcal{I}(\mathtt{j})$

Modified state update of $\mathcal{I}$ at $\mathtt{j}$ of type $\mathtt{z}$ with $d \in \mathcal{D}^{\mathtt{z}}$

$$\mathcal{I}_{\mathtt{j}}^{d}(\mathtt{x}) = \begin{cases} \mathcal{I}(\mathtt{x}) & \mathtt{x} \neq \mathtt{j} \\ d & \mathtt{x} = \mathtt{j} \end{cases}$$

Cf. modified variable assignment

Operational Semantics of Programs

State transformation ρ defines semantics of programs

Same ρ for all programs, so not part of s

- $\rho(\mathtt{x\ =\ t;})(\mathcal{I}) = \mathcal{I}_{\mathtt{x}}^{val_{\mathcal{I},\beta}(t)}$ **(can ignore β)**

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- $\rho(\texttt{pq})(\mathcal{I}) = \rho(\texttt{q})(\rho(\texttt{p})(\mathcal{I}))$, **if** $\rho(\texttt{p})(\mathcal{I})$ **defined, undefined otherwise**

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- $\rho(\mathtt{x} \;=\; \mathtt{t;})(\mathcal{I}) = \mathcal{I}_{\mathtt{x}}^{val_{\mathcal{I},\beta}(t)}$ **(can ignore β)**
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- $\rho(\mathtt{pq})(\mathcal{I}) = \rho(\mathtt{q})(\rho(\mathtt{p})(\mathcal{I}))$**, if** $\rho(\mathtt{p})(\mathcal{I})$ **defined, undefined otherwise**
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Need to have rules for program formulas: but which?

What corresponds to top-level connective in sequential program?

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Rules execute symbolically the first active statement

Proof corresponds to symbolic program execution

Dynamic Logic Calculus

$$\text{CONCATENATE}\ \frac{\Gamma \quad \texttt{==>} \quad \langle \mathtt{p} \rangle \left(\langle \mathtt{q} \rangle \phi \right), \Delta}{\Gamma \quad \texttt{==>} \quad \langle \mathtt{pq} \rangle \phi, \Delta}$$

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Dynamic Logic Examples

Partial correctness assertion **(Hoare formula)**

$$\{\psi\}\ \mathtt{p}\ \{\phi\}$$

If $\mathtt{p}$ is started in a state satisfying ψ and terminates, then its final state satisfies ϕ

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Induction Rule

Motivation

- UNWIND-rule only works if number of loop iterations small & known
- Properties of inductive FO data structures unprovable

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Induction Rule (over natural numbers)

$$\frac{\Gamma \ \texttt{==>}\ [n/0]\,\phi, \Delta \qquad \Gamma, [n/n']\,\phi \ \texttt{==>}\ [n/n'{+}1]\,\phi, \Delta \qquad \Gamma, \forall n.\phi \ \texttt{==>}\ \Delta}{\Gamma \ \texttt{==>}\ \Delta}$$

Where n logical variable, n' constant of type `int` not occurring in Γ, Δ

Induction Rule Example

Definition of $\mathbf{even}$ (unary predicate on `int`):

- **==>** $\mathbf{even}(0)$
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Demo in **`dlIntro/ind.key`**

Quantifying over Program Variables

What if induction hypothesis contains program?

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Solution

Use explicit construct to record state change information

Update $\forall n.(\mathbf{\{}\mathtt{i} := n\mathbf{\}}\langle \mathtt{p}(\mathtt{i})\rangle\phi)$

Explicit State Updates

Updates record computation state in which we evaluate a formula

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Semantics

$$\mathcal{I}, \beta \models \{\mathtt{v} := t\}\phi \quad \textbf{iff} \quad \mathcal{I}_{\mathtt{v}}^{val_{\mathcal{I},\beta}(t)}, \beta \models \phi$$

Semantics identical to assignment, may depend on logical variables in t

Updates work like “lazy” assignments

Updates are not assignments: may contain logical variable

Updates are not equations: change interpretation of non-rigid terms

Computing Effect of Updates (Automatic)

Update followed by program variable | **by logical variable**

$$\{\mathtt{x} := t\}\mathtt{y} \quad \rightsquigarrow \quad \mathtt{y}$$

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Update followed by complex term

$$\{\mathtt{x} := t\}f(t_1, \ldots, t_n) \quad \leadsto \quad f(\{\mathtt{x} := t\}t_1, \ldots, \{\mathtt{x} := t\}t_n)$$

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Update followed by program formula

$$\{\mathtt{x} := t\}(\langle \mathtt{p} \rangle \phi) \quad \rightsquigarrow \quad \{\mathtt{x} := t\}(\langle \mathtt{p} \rangle \phi)$$

unchanged!

Update computation delayed until $\mathtt{p}$ symbolically executed

Assignment Rule Using Updates

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Avoids renaming of program variables

Works as long as t has no side effects (ok in simple DL)

But: rules dealing with programs need to account for updates

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General form of conclusion in rule for symbolic execution

$$\Gamma \ \texttt{==>}\ \mathcal{U}\langle\ \pi\ \ \xi;\ \ \omega\ \rangle \phi, \Delta$$

Prefix Active statement Postfix

Example Proof

```
\programVariables { // program variables in FSym
  int x;
}
\problem {
 \exists int y; (x = y ->  // y logical variable
    \<{while (x > 0) {x = x-1;}}\> true)
      // modal brackets written as \<, \>
}
```

Intuitive Meaning? Satisfiable? Valid?

Demo

`dlIntro/term.key`