# CS:5810 Formal Methods in Software Engineering

## Introduction to Alloy

## Part 1

# Outline

- Introduction to basic Alloy constructs using a simple example of a static model
  - How to define sets, subsets, relations with multiplicity constraints
  - How to use Alloy's quantifiers and predicate forms
- Basic use of the Alloy Analyzer 4 (AA)
  - Loading, compiling, and analyzing a simple Alloy specification
  - Adjusting basic tool parameters
  - Using the visualization tool to view instances of models

# Roadmap

Alloy: Rationale and Use Strategies

- What types of systems have been modeled with Alloy
- What types of questions can AA answer
- What is the purpose of each of the sections in an Alloy specification

Alloy Specifications

- Parameterized conditionals
- Indexed relations
- Graphical representations of Alloy models
- More complex examples

# Alloy --- Why was it created?

## Lightweight

small and easy to use, and capable of expressing common properties tersely and naturally

## Precise

having a simple and uniform mathematical semantics

## Tractable

amenable to efficient and fully automated semantic analysis (within scope limits)

# Alloy --- Comparison

UML

- Has similarities (graphical notation, OCL constraints) but it is neither lightweight, nor precise
- UML includes many modeling notions omitted from Alloy (use-cases, state-charts, code architecture specs)
- Alloy's diagrams and relational navigation are inspired by UML

Z

- Precise, but intractable. Stylized typography makes it harder to work with.
- Z is more expressive than Alloy, but more complicated
- Alloy's set-based semantics is inspired by Z

# Alloy --- What is it used for?

Alloy is a textual modeling language aimed at expressing structural and behaviorals properties of software systems

It is not meant for modeling code architecture (*a la* class diagrams in UML)

But there might be a close relationship between the Alloy specification and an implementation in an OO language

# Alloy --- Example Applications

The Alloy 4 distribution comes with several example models that together illustrate the use of Alloy's constructs.

Examples

- Specification of a distributed spanning tree
- Model of a generic file system
- Model of a generic file synchronizer
- Tower of Hanoi model
- …

# Alloy in General

Alloy is general enough that it can model

- any domain of individuals and
- relations between them

We will then start with a few simple examples that are not necessarily about about software

# Example: Family Structure

We want to…

- Model parent/child relationships as primitive relations
- Model spousal relationships as primitive relations
- Model relationships such as "siblings" as *derived* relations
- Enforce certain biological constraints via 1st-order predicates (e.g., people have only one mother)
- Enforce certain social constraints via 1st-order predicates (e.g., a wife isn't a sibling)
- Confirm or refute the existence of certain derived relationships (e.g., no one has a wife with whom he shares a parent)

# Example: addressBook

An address book for an email client that maintains a mapping from names to addresses

| FriendBook |
|---|
| Ted -> ted@gmail.com<br>Ryan -> ryan@hotmail.com |

| WorkBook |
|---|
| Pilard -> lpilard@uiowa.edu<br>Ryan -> ryan@uiowa.edu |

# Atoms and Relations

In Alloy, everything is built from atoms and relations

An atom is a primitive entity that is

- *indivisible*: it cannot be broken down into smaller parts
- *immutable*: its properties do not change over time
- *uninterpreted*: it does not have any built in property
  (the way numbers do for example)

A relation is a structure that relates atoms. It is a set of tuples, each tuple being a sequence of atoms

# Atoms and Relations: Examples

- Unary relations: a set of names, a set of addresses and a set of books

  Name = {(N0),(N1),(N2)}

  Addr = {(D0),(D1)}

  Book = {(B0),(B1)}

- A binary relation from names to addresses

  address = {(N0,D0),(N1,D1)}

- A ternary relation from books to name to addresses

  addr = {(B0,N0,D0), (B0,N1,D1), (B1,N1,D2)}

Atoms

Tuples

# Relations

Size of a relation: the number of tuples in the relation

Arity of a relation: the number of atoms in each tuple of the relation

- relations with arity 1, 2, and 3 are said to be *unary*, *binary*, and *ternary* relations

Examples:

- relation of arity 1 and size 1: myName = {(N0)}
- relation of arity 2 and size 3: address = {(N0,D0),(N1,D1),(N2,D1))

# Main Components of Alloy Models

- Signatures and Fields
- Predicates and Functions
- Facts
- Assertions
- Commands and scopes

# Signatures and Fields

## Signatures

- Describe classes of entities we want to reason about
- Sets defined in signatures are fixed (dynamic aspects can be modeled by time-dependent relations)

## Fields

- Define relations between signatures

## Simple constraints

- Multiplicities on signatures
- Multiplicities on relations

# Signatures

- A signature introduces a set of atoms

- The declaration

```
sig A {}
```

introduces a set named A

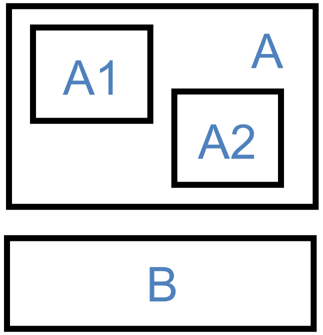
- A set can be introduced as an extension of another; thus

```
sig A1 extends A {}
```

introduces a set A1 that is a subset of A

# Signatures

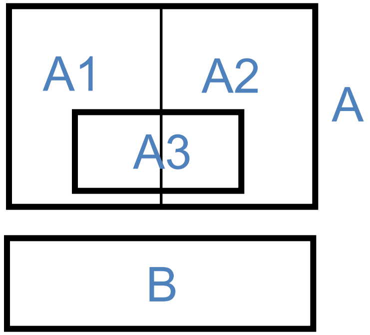
```
sig A {}
sig B {}
sig A1 extends A {}
sig A2 extends A {}
```




- A1 and A2 are extensions of A
- A signature declared independently of any other one is a top-level signature, e.g., A and B
- Extensions of the same signature are mutually disjoint, as are top-level signatures

# Signatures

```
abstract sig A {}
sig B {}
sig A1 extends A {}
sig A2 extends A {}
```




- A signature can be introduced as a subset of another

```
sig A3 in A {}
```

- An abstract signature has no elements except those belonging to its extensions or subsets
- All extensions of an abstract signature A form a partition of A

# Fields

- Relations are declared as fields of signatures
  - Writing

    ```
    sig A {f: e}
    ```

    introduces a relation f of type A x e, where e is an expression denoting a product of signatures)

- Examples: (with signatures A, B, C)
  - Binary Relation:

    ```
    sig A { f1: B }        // f1 is a subset of A x B
    ```

  - Ternary Relation:

    ```
    sig A { f2: B -> C }  // f2 is a subset of A x B x C
    ```

# Example Signatures and Fields

*Family Structure:*

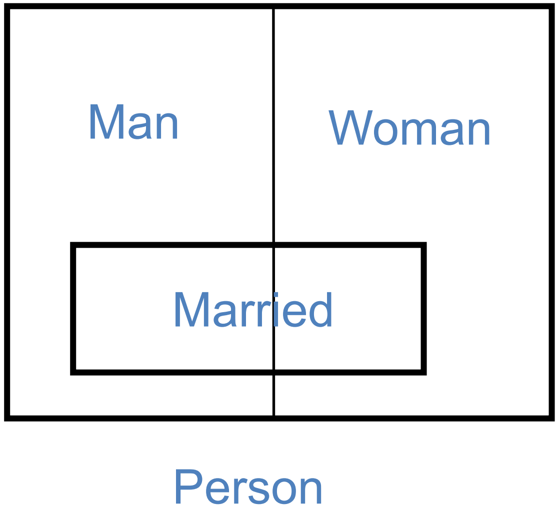
```
abstract sig Person {
    children: Person,
    siblings: Person
}

sig Man, Woman extends Person {}

sig Married in Person {
    spouse: Married
}
```

Fields

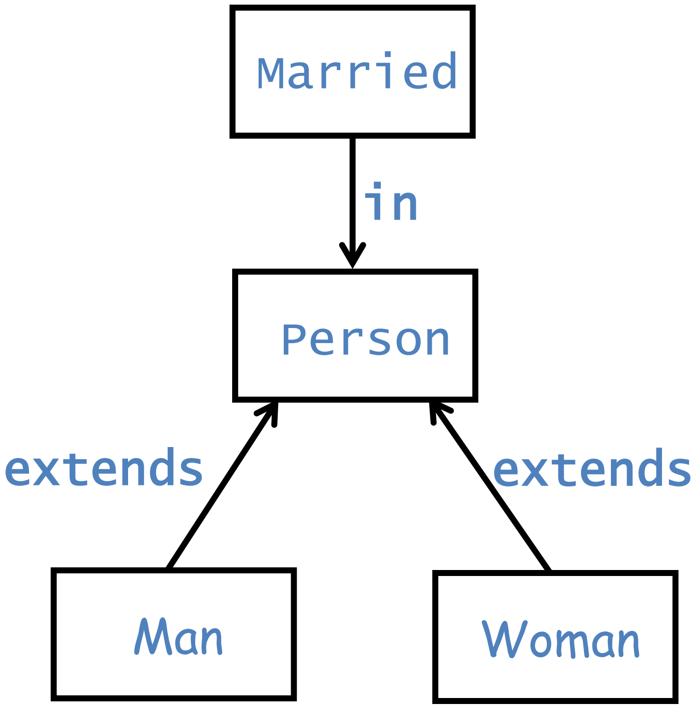
# Example: Family Structure

*Alloy Model*

```
abstract sig Person {}
sig Man extends Person {}
sig Woman extends Person {}
sig Married in Person {}
```




*Graphical Representation*



# Model Instances

The Alloy Analyzer will generate instances of models so that we can see if they match our intentions.
Which of the following are instances of our current model?

```
abstract sig Person {}
sig Man extends Person {}
sig Woman extends Person {}
sig Married in Person {}
```

B. ✗
Person = {(P0),(P1),(P2)}
Man = {(P1),(P2)}
Married = {}
Woman = {(P0),(P1)}

D. ✗
Person = {(P0),(P1)}
Man = {(P0)}
Married = {(P1)}
Woman = {}

A. ✓
Person = {(P0),(P1),(P2)}
Man = {(P1),(P2)}
Married = {}
Woman = {(P0)}

C. ✓
Person = {(P0),(P1),(P2),(P3)}
Man = {(P0),(P1),(P2),(P3)}
Married = {(P2),(P3)}
Woman = {}

E. ✓
Person = {(P0),(P1)}
Man = {(P0)}
Married = {(P1),(P0)}
Woman = {(P1)}

# Example: Family Structure

*Alloy Model with siblings*

```
abstract sig Person {
  siblings: Person
}
sig Man extends Person {}
sig Woman extends Person {}
sig Married in Person {}
```

*siblings is a binary relation*
*it is a subset of Person x Person*

*Example instance*

Person = {(P0), (P1)}
Man = {(P0), (P1)}
Married = {}
Woman = {}

siblings = {(P0,P1), (P1,P0)}

*Intuition: P0 and P1 are siblings*

# Multiplicities

Allow us to constrain the sizes of sets

- A multiplicity keyword placed before a signature declaration constraints the number of element in the signature's set

```
m sig A {}
```

- We can also make multiplicities constraints on fields:

```
sig A {f: m e}
sig A {f: e1 m -> n e2}
```

There are four multiplicities

- `set` : any number
- `some` : one or more
- `lone` : zero or one
- `one` : exactly one

# Multiplicities: Examples

Without multiplicity:

A set of colors, each of which is red, yellow or green

```
abstract sig Color {}
sig Red, Yellow, Green extends Color {}
```

With multiplicity:

An enumeration of colors

```
abstract sig Color {}
one sig Red, Yellow, Green extends Color {}
```

# Multiplicities: Examples

- A file system in which each directory contains any number of objects, and each alias points to exactly one object

```
abstract sig Object {}
sig Directory extends Object {contents: set Object}
sig File extends Object {}
sig Alias in File {to: one Object}
```

redundant

- The default keyword, if omitted, is **`one`**, so:

  `sig A {f: e}` and `sig A {f: one e}`

  are equivalent.

# Multiplicities: Examples

- A book maps names to addresses
  - There is at most one address per Name
  - An address is associated to at least one name

```
sig Name, Addr {}
sig Book {
   addr: Name some -> lone Addr
}
```

# Multiplicities: Examples

- A collection of weather forecasts, each of which has a field weather associating every city with exactly one weather condition

```
sig Forecast {weather: City -> one Weather}
sig City {}
abstract sig Weather {}
one sig Rainy, Sunny, Cloudy extends Weather {}
```

- Instance:

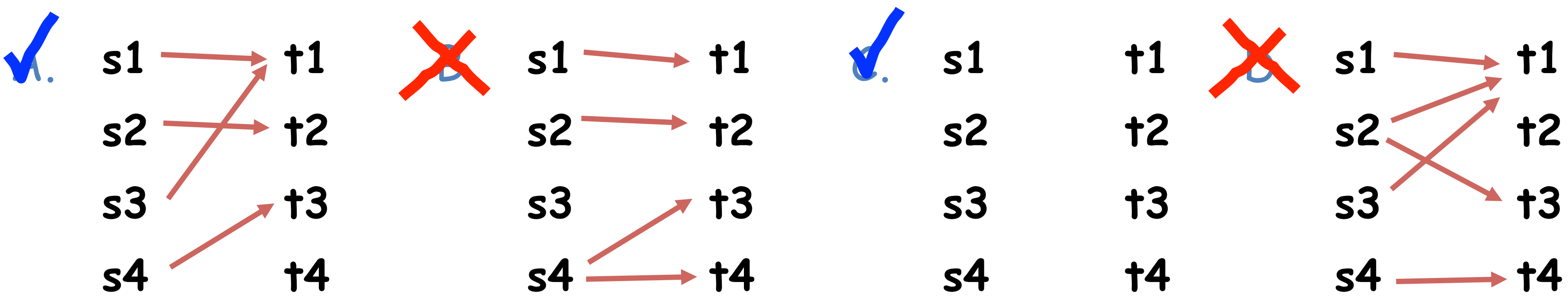
City = {(Iowa City), (Chicago)}
Rainy = {(rainy)}
Sunny = {(sunny)}
Cloudy = {(cloudy)}
Forecast = {(f1), (f2)}
weather = { (f1, Iowa City, rainy), (f1, Chicago, rainy),
(f2, Iowa City, rainy), (f2, Chicago, sunny) }

# Multiplicities and Binary Relations

- `sig S {f: lone T}`
  - says that, for each element s of S, f maps s to at most a single value in T

*Conventional name:* partial function

- Potential instances:

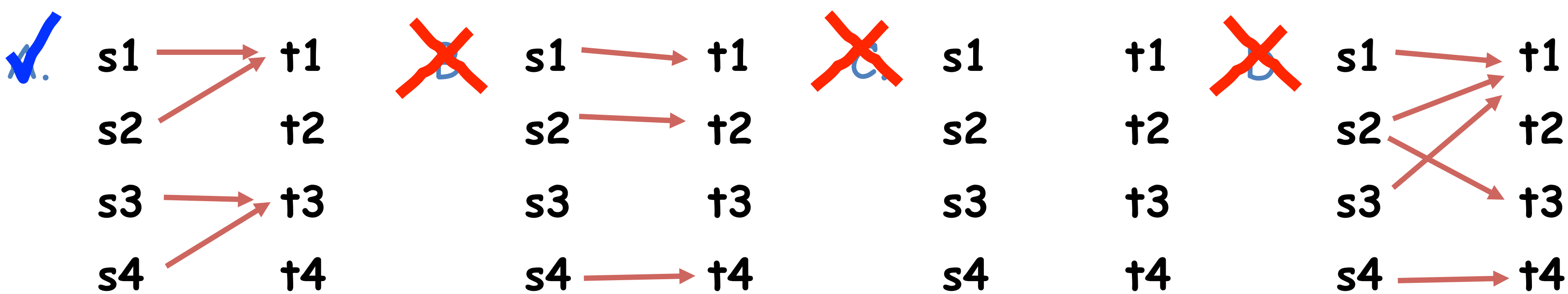


# Multiplicities and Binary Relations

- `sig S {f: one T}`
  - says that, for each element s of S, f maps s to exactly one value in T

*Conventional name:* total function

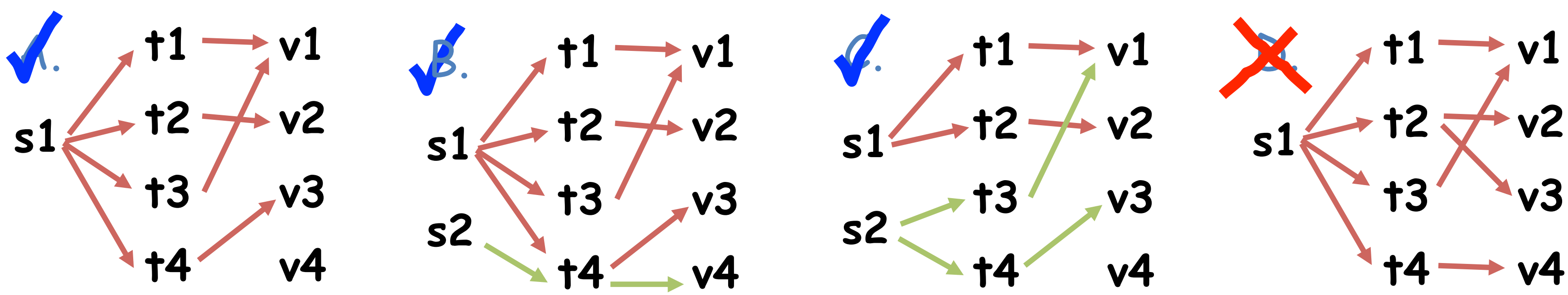
- Potential instances:



# Multiplicities and Ternary Relations

- `sig S {f: T -> one V}`
  - For each element s of S, over the triples that start with s: f maps each T-element to exactly one V-element

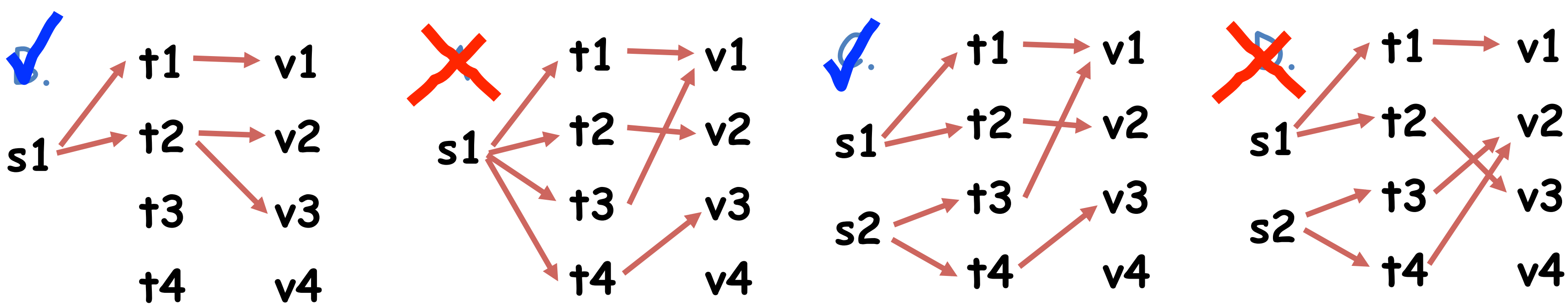
- Potential instances:



# Multiplicities and Ternary Relations

- `sig S {f: T lone -> V}`
  - For each element s of S, over the triples that start with s: f maps at most one T-element to the same V-element

- Potential instances:



# Multiplicities and Relations

- Other kinds of relational structures can be specified using multiplicities

- Examples:
  - `sig S {f: some T}` … `total relation`
  - `sig S {f: set T}` … `partial relation`
  - `sig S {f: T set -> set V}`
  - `sig S {f: T one -> V}`
  - …

# Cardinality Constraints

Multiplicities can also be applied to whole expressions denoting relations

- `some e` e is non-empty
- `no e` e is empty
- `lone e` e has at most one tuple
- `one e` e has exactly one tuple

# Example: Family Structure

- How would you use multiplicities to define the children relation?

```
sig Person {children: set Person}
```

  – Intuition: each person has zero or more children

- How would you use multiplicities to define the spouse relation?

```
sig Married {spouse: one Married}
```

  – Intuition: each married person has exactly one spouse

# Summarizing

*Alloy Model*

```
abstract sig Person {
     children: set Person,
     siblings: set Person
}

sig Man, Woman extends Person {}

sig Married in Person {
     spouse: one Married
}
```

# Exercises

- Start the Alloy Analyzer:
- Load file `family-1.als` from the **Resources** section of the course website
- Execute it
- Analyze the model instance
- Look at the generated instance
- Does it look correct?
- What, if anything, would you change about it?

# Model Instance

```
Instance found:

Person = {Man0,Man1,Man2}
Man = {Man0,Man1,Man2}
Woman = {}
Married = {Man0,Man1,Man2}

children = { (Man0,Man0),(Man0,Man1),
           (Man1,Man0),
           (Man2,Man1), (Man2,Man2)
}
siblings = { (Man0,Man0),(Man0,Man1),
         (Man1,Man0),(Man1,Man2),
         (Man2,Man2)
}
spouse = {(Man1,Man0),(Man0,Man2),(Man2,Man0)}
```

# Man can be his own child ?

```
Instance found:

Person = {Man0,Man1,Man2}
Man = {Man0,Man1,Man2}
Woman = {}
Married = {Man0,Man1,Man2}

children = { (Man0,Man0),(Man0,Man1),
             (Man1,Man0),
             (Man2,Man1), (Man2,Man2)
}
siblings = { (Man0,Man0),(Man0,Man1),
           (Man1,Man0),(Man1,Man2),
           (Man2,Man2)
}
spouse = {(Man1,Man0),(Man0,Man2),(Man2,Man0)}
```

# Multiple Fathers?

```
Instance found:

Person = {Man0,Man1,Man2}
Man = {Man0,Man1,Man2}
Woman = {}
Married = {Man0,Man1,Man2}

children = { (Man0,Man0),(Man0,Man1),
             (Man1,Man0),
             (Man2,Man1), (Man2,Man2)
}
siblings = { (Man0,Man0),(Man0,Man1),
           (Man1,Man0),(Man1,Man2),
           (Man2,Man2)
}
spouse = {(Man1,Man0),(Man0,Man2),(Man2,Man0)}
```

# Self-Siblings ?

```
Instance found:

Person = {Man0,Man1,Man2}
Man = {Man0,Man1,Man2}
Woman = {}
Married = {Man0,Man1,Man2}

children = { (Man0,Man0),(Man0,Man1),
             (Man1,Man0),
             (Man2,Man1), (Man2,Man2)
}
siblings = { (Man0,Man0),(Man0,Man1),
         (Man1,Man0),(Man1,Man2),
         (Man2,Man2)
}
spouse = {(Man1,Man0),(Man0,Man2),(Man2,Man0)}
```

# Asymmetric Siblings ?

```
Instance found:

Person = {Man0,Man1,Man2}
Man = {Man0,Man1,Man2}
Woman = {}
Married = {Man0,Man1,Man2}

children = { (Man0,Man0),(Man0,Man1),
             (Man1,Man0),
             (Man2,Man1), (Man2,Man2)
}
siblings = { (Man0,Man0),(Man0,Man1),
           (Man1,Man0),(Man1,Man2),
           (Man2,Man2)
}
spouse = {(Man1,Man0),(Man0,Man2),(Man2,Man0)}
```

where is (Man2,Man1) ?

# Child-Siblings?

```
Instance found:

Person = {Man0,Man1,Man2}
Man = {Man0,Man1,Man2}
Woman = {}
Married = {Man0,Man1,Man2}

children = { (Man0,Man0),(Man0,Man1),
             (Man1,Man0),
             (Man2,Man1), (Man2,Man2)
}
siblings = { (Man0,Man0),(Man0,Man1),
           (Man1,Man0),(Man1,Man2),
           (Man2,Man2)
}
spouse = {(Man1,Man0),(Man0,Man2),(Man2,Man0)}
```

# Asymmetric Marriage?

```
Instance found:

Person = {Man0,Man1,Man2}
Man = {Man0,Man1,Man2}
Woman = {}
Married = {Man0,Man1,Man2}

children = { (Man0,Man0),(Man0,Man1),
             (Man1,Man0),
             (Man2,Man1), (Man2,Man2)
}
siblings = { (Man0,Man0),(Man0,Man1),
           (Man1,Man0),(Man1,Man2),
           (Man2,Man2)
}
spouse = {(Man1,Man0),(Man0,Man2),(Man2,Man0)}
```

where is (Man0,Man1) ?

# Model Weaknesses

- The model is underconstrained
  - It doesn't match our domain knowledge
  - We can add constraints to enrich the model
- Underconstrained models are common early on in the development process
  - AA gives us quick feedback on weaknesses in our model
  - We can incrementally add constraints until we are satisfied with it

# Adding Constraints

We'd like to enforce the following constraints which are simply matters of biology

- No person can be their own parent (or more generally, their own ancestor)
- No person can have more than one father or mother
- A person's siblings are those with the same parents

# Adding Constraints

- We'd like to enforce the following social constraints
  - The spouse relation is symmetric
  - A man's wife cannot be one of his siblings