CS:5810
Formal Methods in Software Engineering

Introduction to Alloy
Part 1

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Outline

• Introduction to basic Alloy constructs using a simple example of a static model
  – How to define sets, subsets, relations with multiplicity constraints
  – How to use Alloy’s quantifiers and predicate forms

• Basic use of the Alloy Analyzer 4 (AA)
  – Loading, compiling, and analyzing a simple Alloy specification
  – Adjusting basic tool parameters
  – Using the visualization tool to view instances of models
Roadmap

Alloy: Rationale and Use Strategies
– What types of systems have been modeled with Alloy
– What types of questions can AA answer
– What is the purpose of each of the sections in an Alloy specification

Alloy Specifications
– Parameterized conditionals
– Indexed relations
– Graphical representations of Alloy models
– More complex examples
Alloy --- Why was it created?

**Lightweight**

small and easy to use, and capable of expressing common properties tersely and naturally

**Precise**

having a simple and uniform mathematical semantics

**Tractable**

amenable to efficient and fully automated semantic analysis (within scope limits)
Alloy --- Comparison

UML

- Has similarities (graphical notation, OCL constraints) but it is neither lightweight, nor precise
- UML includes many modeling notions omitted from Alloy (use-cases, state-charts, code architecture specs)
- Alloy’s diagrams and relational navigation are inspired by UML

Z

- Precise, but intractable. Stylized typography makes it harder to work with.
- Z is more expressive than Alloy, but more complicated
- Alloy’s set-based semantics is inspired by Z
Alloy --- What is it used for?

Alloy is a textual modeling language aimed at expressing **structural** and **behavioral** properties of software systems.

It is not meant for modeling code architecture (*à la* class diagrams in UML).

But there might be a close relationship between the Alloy specification and an implementation in an OO language.
Alloy --- Example Applications

The Alloy 4 distribution comes with several example models that together illustrate the use of Alloy’s constructs.

Examples

– Specification of a distributed spanning tree
– Model of a generic file system
– Model of a generic file synchronizer
– Tower of Hanoi model
– ...


Alloy in General

Alloy is general enough that it can model
  – any domain of individuals and
  – relations between them

We will then start with a few simple examples that are not necessarily about software
Example: Family Structure

We want to...

• Model parent/child relationships as primitive relations
• Model spousal relationships as primitive relations
• Model relationships such as “siblings” as derived relations
• Enforce certain biological constraints via 1st-order predicates (e.g., people have only one mother)
• Enforce certain social constraints via 1st-order predicates (e.g., a wife isn’t a sibling)
• Confirm or refute the existence of certain derived relationships (e.g., no one has a wife with whom he shares a parent)
An **address book** for an email client that maintains a mapping from **names** to **addresses**

<table>
<thead>
<tr>
<th>FriendBook</th>
<th>WorkBook</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ted -&gt; <a href="mailto:ted@gmail.com">ted@gmail.com</a></td>
<td>Pilard -&gt; <a href="mailto:lpilard@uiowa.edu">lpilard@uiowa.edu</a></td>
</tr>
<tr>
<td>Ryan -&gt; <a href="mailto:ryan@hotmail.com">ryan@hotmail.com</a></td>
<td>Ryan -&gt; <a href="mailto:ryan@uiowa.edu">ryan@uiowa.edu</a></td>
</tr>
</tbody>
</table>
Atoms and Relations

In Alloy, everything is built from atoms and relations

An atom is a primitive entity that is
  – *indivisible*: it cannot be broken down into smaller parts
  – *immutable*: its properties do not change over time
  – *uninterpreted*: it does not have any built in property
    (the way numbers do for example)

A relation is a structure that relates atoms. It is a set of tuples, each tuple being a sequence of atoms
Atoms and Relations: Examples

- **Unary relations**: a set of names, a set of addresses and a set of books
  
  \[\text{Name} = \{(N0),(N1),(N2)\}\]
  \[\text{Addr} = \{(D0),(D1)\}\]
  \[\text{Book} = \{(B0),(B1)\}\]

- **A binary relation** from names to addresses
  
  \[\text{address} = \{(N0,D0),(N1,D1)\}\]

- **A ternary relation** from books to name to addresses
  
  \[\text{addr} = \{(B0,N0,D0),(B0,N1,D1),(B1,N1,D2)\}\]
Relations

Size of a relation: the number of tuples in the relation

Arity of a relation: the number of atoms in each tuple of the relation

– relations with arity 1, 2, and 3 are said to be unary, binary, and ternary relations

Examples:

– relation of arity 1 and size 1: myName = {((N0))}
– relation of arity 2 and size 3: address = {((N0,D0),(N1,D1), (N2,D1))}
Main Components of Alloy Models

• Signatures and Fields
• Predicates and Functions
• Facts
• Assertions
• Commands and scopes
Signatures and Fields

Signatures

– Describe classes of entities we want to reason about
– Sets defined in signatures are fixed (dynamic aspects can be modeled by time-dependent relations)

Fields

– Define relations between signatures

Simple constraints

– Multiplicities on signatures
– Multiplicities on relations
Signatures

- A signature introduces a set of atoms

- The declaration
  
  ```
  sig A {}
  ```
  introduces a set named A

- A set can be introduced as an extension of another; thus
  
  ```
  sig A1 extends A {}
  ```
  introduces a set A1 that is a subset of A
Signatures

\[
\begin{align*}
\text{sig } A & \text{ {} } \\
\text{sig } B & \text{ {} } \\
\text{sig } A_1 & \text{ extends } A \text{ {} } \\
\text{sig } A_2 & \text{ extends } A \text{ {} }
\end{align*}
\]

- A1 and A2 are extensions of A
- A signature declared independently of any other one is a top-level signature, e.g., A and B
- Extensions of the same signature are mutually disjoint, as are top-level signatures
Signatures

abstract sig A {}
sig B {}
sig A1 extends A {}
sig A2 extends A {}

• A signature can be introduced as a **subset** of another
  sig A3 in A {}

• An **abstract signature** has no elements except those belonging to its extensions or subsets

• All extensions of an abstract signature A form a **partition** of A
Fields

• Relations are declared as fields of signatures
  – Writing
    \[ \text{sig } A \{ f: e \} \]
    introduces a relation \( f \) of type \( A \times e \), where \( e \) is an expression denoting a product of signatures

• Examples: (with signatures \( A, B, C \))
  – Binary Relation:
    \[ \text{sig } A \{ f1: B \} \]  // \( f1 \) is a subset of \( A \times B \)
  – Ternary Relation:
    \[ \text{sig } A \{ f2: B \rightarrow C \} \]  // \( f2 \) is a subset of \( A \times B \times C \)
Example Signatures and Fields

Family Structure:

abstract sig Person {
  children: Person,
  siblings: Person
}

sig Man, Woman extends Person {}

sig Married in Person {
  spouse: Married
}

Fields
Example: Family Structure

**Alloy Model**

```alloy
abstract sig Person {}
sig Man extends Person {}
sig Woman extends Person {}
sig Married in Person {}
```

**Graphical Representation**

```
Person
    `extends` Man
    `extends` Woman
    `in` Married
```

```
Man

Woman

Married
```

```
Person
    `extends` Man
    `extends` Woman
    `in` Married
```

```
Man

Woman

Married
```
Model Instances

The Alloy Analyzer will generate instances of models so that we can see if they match our intentions. Which of the following are instances of our current model?

- abstract sig Person {}
sig Man extends Person {}
sig Woman extends Person {}
sig Married in Person {}

- Person = {(P0),(P1),(P2)}
  Man = {(P1),(P2)}
  Married = {}
  Woman = {(P0),(P1)}

- Person = {(P0),(P1),(P2),(P3)}
  Man = {(P0),(P1),(P2),(P3)}
  Married = {(P2),(P3)}
  Woman = {}

- Person = {(P0),(P1)}
  Man = {(P0)}
  Married = {}
Example: Family Structure

**Alloy Model with siblings**

```alloy
abstract sig Person {
  siblings: Person
}

sig Man extends Person {}
sig Woman extends Person {}
sig Married in Person {}
```

*siblings is a binary relation*
*it is a subset of Person x Person*

**Example instance**

- Person = {(P0), (P1)}
- Man = {(P0), (P1)}
- Married = {}
- Woman = {}
- siblings = {(P0,P1), (P1,P0)}

*Intuition: P0 and P1 are siblings*
Multiplicities

Allow us to constrain the sizes of sets

- A multiplicity keyword placed before a signature declaration constraints the number of element in the signature’s set

  \[
  m \text{ sig } A \ {}\ {}
  \]

- We can also make multiplicities constraints on fields:

  \[
  \text{sig} \ A \ \{f: \ m \ e\}
  \]

  \[
  \text{sig} \ A \ \{f: \ e1 \ m \rightarrow n \ e2\}
  \]

There are four multiplicities

- set : any number
- some : one or more
- lone : zero or one
- one : exactly one
Multiplicities: Examples

Without multiplicity:

A set of colors, each of which is red, yellow or green

```plaintext
abstract sig Color {}
sig Red, Yellow, Green extends Color {}
```

With multiplicity:

An enumeration of colors

```plaintext
abstract sig Color {}
one sig Red, Yellow, Green extends Color {}
```
Multiplicities: Examples

- A file system in which each directory contains any number of objects, and each alias points to exactly one object

```plaintext
abstract sig Object {}
sig Directory extends Object {contents: set Object}
sig File extends Object {}
sig Alias in File {to: one Object}
```

- The default keyword, if omitted, is `one`, so:

  ```plaintext
  sig A {f: e} and sig A {f: one e}
  ```

  are equivalent.
Multiplicities: Examples

- A book maps names to addresses
  - There is at most one address per Name
  - An address is associated to at least one name

```
sig Name, Addr {}
sig Book {
    addr: Name some -> lone Addr
}
```
Multiplicities: Examples

- A collection of weather forecasts, each of which has a field weather associating every city with exactly one weather condition

\[
\text{sig } \text{Forecast} \ {\text{weather}}: \text{City} \rightarrow \text{one Weather} \\
\text{sig } \text{City} \ {} \\
\text{abstract sig } \text{Weather} \ {} \\
\text{one sig } \text{Rainy, Sunny, Cloudy extends Weather} \ {} \\
\]

- Instance:

\[
\text{City} = \{(\text{Iowa City}), (\text{Chicago})\} \\
\text{Rainy} = \{(\text{rainy})\} \\
\text{Sunny} = \{(\text{sunny})\} \\
\text{Cloudy} = \{(\text{cloudy})\} \\
\text{Forecast} = \{(\text{f1}), (\text{f2})\} \\
\text{weather} = \{(\text{f1, Iowa City, rainy}), (\text{f1, Chicago, rainy}), (\text{f2, Iowa City, rainy}), (\text{f2, Chicago, sunny})\}
\]
Multiplicities and Binary Relations

- **sig S \{f: \text{ lone } T\}**
  - says that, for each element \( s \) of \( S \), \( f \) maps \( s \) to at most a single value in \( T \)

*Conventional name:* partial function

- Potential instances:
Multiplicities and Binary Relations

- \textbf{sig } S \{ f: \text{ one } T \}
  
  - says that, for each element \( s \) of \( S \), \( f \) maps \( s \) to \textbf{exactly one} value in \( T \)
  
  \textit{Conventional name:} total function

- Potential instances:

  \begin{align*}
  &A. & s_1 & \rightarrow & t_1 & \text{ } & s_2 & \rightarrow & t_2 & \text{ } & s_3 & \rightarrow & t_3 & \text{ } & s_4 & \rightarrow & t_4 \\
  &B. & s_1 & \rightarrow & t_1 & \text{ } & s_2 & \rightarrow & t_2 & \text{ } & s_3 & \rightarrow & t_3 & \text{ } & s_4 & \rightarrow & t_3 \\
  &C. & s_1 & \rightarrow & t_1 & \text{ } & s_2 & \rightarrow & t_2 & \text{ } & s_3 & \rightarrow & t_3 & \text{ } & s_4 & \rightarrow & t_4 \\
  &D. & s_1 & \rightarrow & t_1 & \text{ } & s_2 & \rightarrow & t_2 & \text{ } & s_3 & \rightarrow & t_3 & \text{ } & s_4 & \rightarrow & t_4
  \end{align*}
Multiplicities and Ternary Relations

• \textbf{sig }S \{f: T \rightarrow \text{ one } V\} \\
  - For each element \(s\) of \(S\), over the triples that start with \(s\): \(f\) maps each \(T\)-element to \textbf{exactly} one \(V\)-element

• Potential instances:
Multiplicities and Ternary Relations

• \( \text{sig } S \ \{ f: T \rightarrow V \} \)
  – For each element \( s \) of \( S \), over the triples that start with \( s \):
    \( f \) maps at most one \( T \)-element to the same \( V \)-element

• Potential instances:
Multiplicities and Relations

• Other kinds of relational structures can be specified using multiplicities

• Examples:
  – \texttt{sig S \{f: some T\} } \ldots \text{ total relation}
  – \texttt{sig S \{f: set T\} } \ldots \text{ partial relation}
  – \texttt{sig S \{f: T set -> set V\}}
  – \texttt{sig S \{f: T one -> V\}}
  – …
Cardinality Constraints

Multiplicities can also be applied to whole expressions denoting relations

- **some** $e$  e is non-empty
- **no** $e$  e is empty
- **lone** $e$  e has at most one tuple
- **one** $e$  e has exactly one tuple
Example: Family Structure

• How would you use multiplicities to define the children relation?

  \text{sig Person} \ \{\text{children: set Person}\}

  \quad \text{– Intuition: each person has zero or more children}

• How would you use multiplicities to define the spouse relation?

  \text{sig Married} \ \{\text{spouse: one Married}\}

  \quad \text{– Intuition: each married person has exactly one spouse}
Summarizing

Alloy Model

abstract sig Person {
    children: set Person,
    siblings: set Person
}

sig Man, Woman extends Person {}

sig Married in Person {
    spouse: one Married
}
Exercises

• Start the Alloy Analyzer:
• Load file family-1.als from the Resources section of the course website
• Execute it
• Analyze the model instance
• Look at the generated instance
• Does it look correct?
• What, if anything, would you change about it?
Model Instance

Instance found:

Person = {Man0, Man1, Man2}
Man = {Man0, Man1, Man2}
Woman = {}
Married = {Man0, Man1, Man2}

children = { (Man0, Man0), (Man0, Man1),
             (Man1, Man0),
             (Man2, Man1), (Man2, Man2) }

siblings = { (Man0, Man0), (Man0, Man1),
             (Man1, Man0), (Man1, Man2),
             (Man2, Man2) }

spouse = {(Man1, Man0), (Man0, Man2), (Man2, Man0)}
Man can be his own child?

Instance found:

Person = {Man0, Man1, Man2}
Man = {Man0, Man1, Man2}
Woman = {}
Married = {Man0, Man1, Man2}

children = {
  (Man0, Man0),
  (Man0, Man1),
  (Man1, Man0),
  (Man2, Man1),
  (Man2, Man2)
}

siblings = {
  (Man0, Man0),
  (Man0, Man1),
  (Man1, Man0),
  (Man1, Man2),
  (Man2, Man2)
}

spouse = {(Man1, Man0), (Man0, Man2), (Man2, Man0)}
Multiple Fathers?

Instance found:

Person = {Man0, Man1, Man2}
Man = {Man0, Man1, Man2}
Woman = {}
Married = {Man0, Man1, Man2}

children = {(Man0, Man0), (\textbf{Man0}, Man1),
             (Man1, Man0),
             (\textbf{Man2}, Man1), (Man2, Man2)}

siblings = {(Man0, Man0), (Man0, Man1),
             (Man1, Man0), (Man1, Man2),
             (Man2, Man2)}

spouse = {(Man1, Man0), (Man0, Man2), (Man2, Man0)}}
Self-Siblings?

Instance found:

Person = \{\text{Man0, Man1, Man2}\}
Man = \{\text{Man0, Man1, Man2}\}
Woman = \{
Married = \{\text{Man0, Man1, Man2}\}

children = \{
    (\text{Man0, Man0}), (\text{Man0, Man1}), 
    (\text{Man1, Man0}), 
    (\text{Man2, Man1}), (\text{Man2, Man2})
\}

siblings = \{
    (\text{Man0, Man0}), (\text{Man0, Man1}), 
    (\text{Man1, Man0}), (\text{Man1, Man2}), 
    (\text{Man2, Man2})
\}

spouse = \{(\text{Man1, Man0}), (\text{Man0, Man2}), (\text{Man2, Man0})\}
Asymmetric Siblings?

Instance found:

Person = {Man0, Man1, Man2}
Man = {Man0, Man1, Man2}
Woman = {}
Married = {Man0, Man1, Man2}

children = { (Man0, Man0), (Man0, Man1),
             (Man1, Man0),
             (Man2, Man1), (Man2, Man2) }

siblings = { (Man0, Man0), (Man0, Man1),
             (Man1, Man0), (Man1, Man2),
             (Man2, Man2) }  

spouse = { (Man1, Man0), (Man0, Man2), (Man2, Man0) }

where is (Man2, Man1)?
Child-Siblings?

Instance found:

Person = \{\text{Man0, Man1, Man2}\}
Man = \{\text{Man0, Man1, Man2}\}
Woman = \{
\}
Married = \{\text{Man0, Man1, Man2}\}

\text{children} = \{
(\text{Man0, Man0}),
(\text{Man0, Man1}),
(\text{Man1, Man0}),
(\text{Man2, Man1}),
(\text{Man2, Man2})
\}
\text{siblings} = \{
(\text{Man0, Man0}),
(\text{Man0, Man1}),
(\text{Man1, Man0}),
(\text{Man1, Man2}),
(\text{Man2, Man2})
\}
\text{spouse} = \{(\text{Man1, Man0}), (\text{Man0, Man2}), (\text{Man2, Man0})\}
Asymmetric Marriage?

Instance found:

Person = \{\text{Man0, Man1, Man2}\}
Man = \{\text{Man0, Man1, Man2}\}
Woman = \{
Married = \{\text{Man0, Man1, Man2}\}

children = \{(\text{Man0, Man0}), (\text{Man0, Man1}),
(\text{Man1, Man0}),
(\text{Man2, Man1}), (\text{Man2, Man2})\}

siblings = \{(\text{Man0, Man0}), (\text{Man0, Man1}),
(\text{Man1, Man0}), (\text{Man1, Man2}),
(\text{Man2, Man2})\}

spouse = \{(\text{Man1, Man0}), (\text{Man0, Man2}), (\text{Man2, Man0})\}

where is (\text{Man0, Man1})?
Model Weaknesses

• The model is underconstrained
  – It doesn’t match our domain knowledge
  – We can add constraints to enrich the model

• Underconstrained models are common early on in the development process
  – AA gives us quick feedback on weaknesses in our model
  – We can incrementally add constraints until we are satisfied with it
Adding Constraints

We’d like to enforce the following constraints which are simply matters of biology

- No person can be their own parent (or more generally, their own ancestor)

- No person can have more than one father or mother

- A person’s siblings are those with the same parents
Adding Constraints

• We’d like to enforce the following social constraints

  – The spouse relation is symmetric

  – A man’s wife cannot be one of his siblings