CS:5620 Homework 3, Fall 2016

Due in class on Tue, 10/11

Collaboration: You are welcome to form groups of size 2 and work on your homeworks in groups. Of course, you are not *required* to work in groups. Every group should make one submission and names of both group members should appear on the submission and both students in a group will receive the same score. Other than the TA and the professor, you can only discuss homework problems with your group partner. Collaboration can be positive because talking to someone else about these problems can help to clarify your ideas and you will also (hopefully) get to hear about different ways of thinking about the problem. On the other hand, collaboration can be negative if one member of the group rides on work being done by the other member – please avoid this situation. If your solutions are (even partly) based on material other than what has been posted on the course website, you should clearly acknowledge your outside sources.
Late submissions: No late submissions are permitted. You will receive no points for your submission if your submission is not turned in at the beginning of class on the due date.
Evaluation: Your submissions will be evaluated on correctness *and* clarity. Correctness is of course crucial, but how clearly you communicate your ideas is also quite important.
Who submits what: Undergraduate students are required to solve Problems 1-4(a) and graduate students are required to solve Problems 2-5.

1. Here is a problem on k-ary c-colorings that we studied in the proof of Linial's $\Omega(\log^* n)$ lower bound on 3-coloring oriented cycles.

   (a) Describe a 1-ary n-coloring function.

   (b) Define $A(x_1, x_2) = 1 + (x_1 + x_2) \bmod c$ for any $1 \leq x_1, x_2 \leq n$. Is this a 2-ary c-coloring function? Justify your answer.

   (c) Fix $n = 10$ and $c = 3$. Start with the function $A(\cdot, \cdot)$ defined in (b) and follow the construction in the lower bound proof for 3-coloring oriented graphs (discussed in class and available as a reading on the course webpage) and write down the values of $B(1)$ and $B(2)$.

2. In this problem, you will show an $\Omega(\log^* n)$ lower bound for the MIS problem.

   (a) Design a deterministic $O(1)$-round algorithm in the Congest model that takes as input an MIS of an oriented cycle and outputs a 3-coloring. In other words, when the algorithm starts, each node in the given MIS knows that it is in the MIS and the remaining nodes know that they are not in the MIS.

   (b) The algorithm you designed in (a) can be viewed as a *distributed reduction* from the 3-coloring problem to the MIS problem. Specifically, the algorithm in part (a) shows that if there is an algorithm, running in T rounds, that solves the MIS problem on an oriented cycle in the Congest model (respectively, Local model), then there is a 3-soloring algorithm that runs in $T + O(1)$ rounds in the Congest model (respectively, Local model). Use this to argue that there cannot exist a deterministic $o(\log^* n)$-round algorithm in the Local model for the MIS problem on an oriented cycle.

3. Consider Problem 4 in HW2 and design a randomized algorithm for this problem running in $O(\Delta + \log n)$ rounds in expectation in the Congest model.

4. *Time division multiple access (TDMA)* is a method for different nodes in a distributed system to access a shared physical medium. For example, for nodes in a wireless network, their *radio frequency channel* is the shared medium that they all need to access to be able to communicate. TDMA is also used in other settings such as bus networks, where different nodes need to access a shared bus in order to communicate. In TDMA, time is

partitioned into time slots and each node has its own time slot during which it uses the shared medium.

Consider the situation in which there are n identical[1] nodes in a wireless network, each of which needs to send a message to a base station. The base station is within the transmission range of the wireless nodes, but is possible that not all pairs of wireless nodes are in each others' transmission ranges. The wireless nodes have access to a single radio frequency channel and therefore if two or more of these nodes transmit their message at the same time, the base station hears a "collision," but does not receive any of the transmitted messages. More precisely, a base station has the ability to distinguish among three situations: (i) it hears no transmission, (ii) it hears a "collision", and (iii) it hears a message. The base station can also transmit, but it also uses the same radio frequency channel as the wireless nodes. Therefore, if the base station is transmitting at the same time as a wireless node, it will hear a collision.

Now notice that n nodes have to send messages to the base station and no two nodes can transmit at the same time. So at least n time slots are needed for all the messages to reach the base station. The problem is how should the wireless nodes and the base station coordinate transmissions so that it does not take too many time slots for all messages to reach the base station.

(a) Use ideas you have learned from our discussion of Luby's MIS algorithm to design a randomized algorithm that uses $O(n)$ times slots in expectation to ensure that the messages from all n wireless nodes reach the base station. You can assume that nodes initially know n.

(b) Prove that your algorithm runs in expected $O(n)$ rounds.

5. Euler's Theorem tells us that an n-node *planar graph* has at most $3n - 6$ edges. This fact along with ideas you explored in Problem 5, HW2 can be used to design a *deterministic* $O(\log n)$-round MIS algorithm for planar graphs. Describe this algorithm. You do not need to express your algorithm in pseudocode – plain English will do. But make sure it is clear and well organized. Also, explain why your algorithm runs in $O(\log n)$ rounds. You do not have to prove correctness of your algorithm.

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[1] Here "identical" means that nodes do not have IDs.