Assume that \( P(v) \) is initially set to \( \{1, 2, \ldots, 2\Delta\} \) for all nodes \( v \). This assumption is just for this “warm-up” analysis; in the actual algorithm, \( P(v) = \{1, 2, \ldots, \text{degree}(v) + 1\} \) initially for all nodes \( v \). Note that this assumption implies that \( |P(v)| \geq \Delta \) for all nodes \( v \), throughout the algorithm. Fix a round \( i \) and let \( u \) be an arbitrary node that has not been colored after the first \( i - 1 \) rounds. Let \( X_u \) denote the event that \( u \) is colored in round \( i \). We will show that \( \Pr(X_u) \geq 1/4 \).

Let \( W_{c,u} \) denote the event that node \( u \) has selected color \( c \), as a “tentative” color. Note that \( \Pr(W_{c,u}) = 1/|P(u)| \) if \( c \in P(u) \) and otherwise \( \Pr(W_{c,u}) = 0 \).

\[
Pr(X_u) = Pr(\exists c \in P(u) : W_{c,u} \land \forall v \in N(u) : \overline{W_{c,v}})
\]
\[
= \sum_{c \in P(u)} Pr(W_{c,u} \land \forall v \in N(u) : \overline{W_{c,v}})
\]
\[
= \sum_{c \in P(u)} Pr(W_{c,u}) \cdot Pr(\forall v \in N(u) : \overline{W_{c,v}})
\]
\[
= \frac{1}{|P(u)|} \sum_{c \in P(u)} Pr(\forall v \in N(u) : \overline{W_{c,v}})
\]
\[
= \frac{1}{|P(u)|} \sum_{c \in P(u)} \prod_{v \in N(u)} Pr(\overline{W_{c,v}}) \tag{1}
\]

Now we focus on finding a lower bound for \( Pr(\overline{W_{c,v}}) \). If \( c \not\in P(v) \) then \( Pr(\overline{W_{c,v}}) = 1 \). If \( c \in P(v) \) then \( Pr(\overline{W_{c,v}}) = 1 - 1/|P(v)| \). Therefore, independent of whether \( c \in P(v) \), we see that \( Pr(\overline{W_{c,v}}) \geq (1 - 1/|P(v)|) \). Since \( |P(v)| \geq \Delta \), this implies that \( Pr(\overline{W_{c,v}}) \geq (1 - 1/\Delta) \).

Plugging this lower bound into the right hand side of (1) we get

\[
Pr(X_u) \geq \frac{1}{|P(u)|} \sum_{c \in P(u)} \prod_{v \in N(u)} \left(1 - \frac{1}{\Delta}\right) \tag{2}
\]

Since \( |N(u)| \leq \Delta \),

\[
\prod_{v \in N(u)} \left(1 - \frac{1}{\Delta}\right) \geq \left(1 - \frac{1}{\Delta}\right)^\Delta.
\]

Plugging this into (2), we get

\[
Pr(X_u) \geq \frac{1}{|P(u)|} \sum_{c \in P(u)} \left(1 - \frac{1}{\Delta}\right)^\Delta = \left(1 - \frac{1}{\Delta}\right)^\Delta.
\]

Now we use the fact that \((1 - 1/x)^x \geq 1/4\) for all \( x \geq 2 \) to obtain the result \( Pr(X_u) \geq 1/4 \) for \( \Delta \geq 2 \).