CS:5340 Homework 6

Due: Tue, 11/28

Notes: (a) Any problem numbers mentioned in the handout refer to problems in the textbook, by Arora and Barak. (b) It is possible that solutions to some of these problems are available to you via other theory of computation books or on-line lecture notes, etc. If you use any such sources, please acknowledge these in your homework *and* present your solutions in your own words. You will benefit most from the homework, if you seriously attempt each problem on your own first, before seeking other sources. (c) As mentioned in the syllabus, it is okay to form groups of two in solving and submitting homework solutions. But, my advice from (b) still applies: you will benefit most from the homework, if you seriously attempt each problem on your own first, before seeking help from your group partner. (d) Discussing these problems with any of your classmates is okay, provided you and your classmates are not being too specific about solutions. In any case, make sure that you take no written material away from these discussions *and* (as in (b)) you present your solutions in your own words. When discussing homework with classmates please be aware of guidelines on "Academic Dishonesty" as mentioned in the course syllabus.

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1. Show that $\Sigma_2^P = NP^{\text{IndSet}}$. Recall that IndSet is the decision problem whose input is a graph G and a positive integer k and we want to know if G contains an independent set of size at least k.

2. Suppose the cities in a Euclidean instance of the *Travelling Salesman Problem (TSP)* are the vertices of a convex polygon. Then the optimum tour is easy to find – it is just the perimeter of the convex polygon. But more importantly for us, the input instance has the *master property*, i.e., there is a tour (the "master tour") such that the optimum tour of any subset of cities is obtained by simply omitting from the master tour the cities not in the subset.

   Show that deciding whether a given instance of the TSP has the master property is in Σ_2^P. (Here we are referring to arbitrary TSP instances, not just Eucldean instances.)

3. Show that if 3Sat is polynomial-time reducible to $\overline{\text{3Sat}}$ then $PH = NP$.

4. Show that $\Pi_2^P = coNP^{SAT}$.

5. Problem 5.9.
