CS:4310 Midterm Exam, Fall 2021

Your answers will be graded primarily for correctness, but clarity, precision, and conciseness will also be important.

1. Use the recursion tree method to solve the recurrence

$$K(n) = K(n/2) + 2K(n/3) + 3K(n/4) + n^2.$$

   Show all your work.

2. This problem is about QuickSort. Recall that the *median-of-three* heuristic examines the first, last, and middle element of the array, and uses the median of those three elements as a quicksort pivot. Prove that QuickSort with the median-of-three heuristic requires $\Omega(n^2)$ time to sort an array of size n in the worst case. Specifically, for any integer n, describe a permutation of the integers 1 through n, such that in every recursive call to median-of-three-quicksort, the pivot is always the second smallest element of the array.

   In general, designing this permutation requires intimate knowledge of the Partition subroutine. To make things simpler, assume that the Partition subroutine is stable,meaning it preserves the existing order of all elements smaller than the pivot, and it preserves the existing order of all elements larger than the pivot. If we make this assumption, then you don't have to be concerned about the specific details of Partition.

3. Let $G = (V, E)$ be a graph. A subset $S \subseteq V$ of vertices is called a *dominating set* if every vertex in G is either in S or has a neighbor in S. The *Minimum Dominating Set (MDS)* problem takes as input a graph G and outputs a dominating set of minimum size.

   Present the recurrences for a dynamic programming algorithm that solves MDS on a rooted tree.

4. Suppose we are given a set L of n line segments in the plane, where each segment has one endpoint on the vertical line $y = 0$ and one endpoint on the vertical line $y = 1$, and all $2n$ endpoints are distinct. Describe and analyze a dynamic programming algorithm to compute the largest subset of L in which no pair of segments intersects. Follow the 6-step approach of constructing dynamic programming algorithms described in class.