Fall 2021 CS: 4310 Homework 6

Let $G = (V, E)$ be a directed graph. For any pair of vertices $u, v \in V$, we say v is *reachable* from u is there is a *directed* path from u to v in G. Note that v being reachable from u does not imply that u is reachable from v. For any vertex $v \in V$, let T_v denote the set of all vertices reachable from v.

Problem: Maximum Directed Coverage (MDC) Given a directed graph $G = (V, E)$ and a positive integer B, find a subset $S \subseteq V$ of vertices of size B such that the *reachability of* S, defined as

$$\left| \bigcup_{v \in S} T_v \right|$$

is maximized.

You may have noticed that MDC is just the Maximum Coverage problem, except that here the sets T_v are provided implicitly by reachability relation in a directed graph. Your task for this homework is to write a program that reads as input a graph $G = (V, E)$, a positive integer B and uses the greedy algorithm (discussed in class) to construct a vertex set S_{Gr} of size B such as $|S_{Gr}| \geq (1 - 1/e) \cdot |S^*|$, where S^* is an optimal solution to MDC.

Your program should contain at least the following functions:

1. A function that reads a directed graph from an input file and stored the graph in an adjacency list data structure.
2. A function that takes as arguments a directed graph G and a vertex v in G and returns T_v, the set of vertices reachable from v. This function will essentially do a graph traversal on G with source v.
3. A function that takes as arguments a directed graph G and a positive integer B and implements the greedy algorithm for MDC, which is a $(1 - 1/e)$-approximation algorithm.

The input file is best explained with the following example:

```
3
5 7
2 1
2 0
0 1
1 3
3 0
3 4
4 1
```

This input file tells us that $B = 3$ and the directed graph has 5 vertices and 7 edges. The directed edges of the graph are specified, one per line, starting in line 3 of the input file. Labels 0 through $n - 1$ are used to specify the vertices of an n-vertex graph.

More details on what exactly to submit will be provided in a few days.