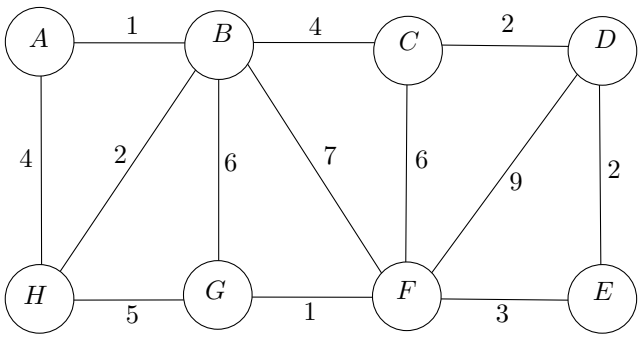
CS:3330 Quiz 4, Spring 2018

1. Consider the edge-weighted graph shown below. We have partially run some MST algorithm and it has determined that edges $\{A, B\}$, $\{B, H\}$, $\{G, F\}$, and $\{D, E\}$ belong to an MST. Using the *cut property* of MSTs figure out all the edges that can safely be added to this partial MST. In other words, let $X = \Big\{\{A, B\}, \{B, H\}, \{G, F\}, \{D, E\}\Big\}$ and your task is to write down all edges $e \notin X$ such $X \cup \{e\}$ is part of some MST. For each edge e that you write down, also write down the corresponding cut $(S, V - S)$ that helped you decide that e is safe.



Solution: There are three edges e, each of which can be added to X, such that $X \cup \{e\}$ is still a part of the MST. Below, for each edge e, I provide cuts $(S, V - s)$ such that e is a lightest edge crossing this cut.

$e = \{B, C\}$; $S = \{A, B, H\}$.
$e = \{E, F\}$; $S = \{F, G\}$ or $S = \{A, B, F, G, H\}$.
$e = \{C, D\}$; $S = \{C\}$ or $S = \{A, B, C, F, G, H\}$, or $S = \{A, B, C, H\}$.

2. Consider an input to the SetCover problem with $|B| = 10000$ (i.e., we need to "cover" a set B with 10,000 elements). Suppose that somehow we know that B can be optimally covered by using 20 sets. Further suppose that after 5 iterations of the greedy algorithm for SetCover, it has picked sets that cover 7000 elements.

   (a) Fill in the blank in the following statement:

   > Based on this information, it is possible to say that in the next iteration, the greedy algorithm will pick a set of size at least _____.

   (b) Explain your answer for (a) in 1-2 sentences.

Solution: (a) 150, (b) After 5 iterations of greedy 3000 elements in B remain uncovered. The optimal solution covers these elements with 20 sets and therefore there is at least one set among those picked by the optimal solution that covers at least $3000/20 = 150$ elements in B. Therefore in the next iteration, the greedy algorithm will pick a set that covers at least 150 elements in B.