CS:3330 Quiz 1, Spring 2017 (Version 1)

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1. Which of the following statements is true about the two functions $f_1(n) = 10n^2 + 3n$ and $f_2(n) = 52 - 32n + 10n^2$?

   (a) $f_1(n)$ grows asymptocially faster than $f_2(n)$

   (b) $f_1(n)$ and $f_2(n)$ asymptotically grow at the same rate

   (c) $f_2(n)$ grows asymptotically faster than $f_1(n)$

   Answer (b). This is because $f_1(n)$ and $f_2(n)$ are both polynomials quadratic polynomials so (a power of a logarithm) asymptotically they grow at the same rate. Note that we are looking at *growth* of functions and not the specific values of the functions for very large n.

2. Which of the following statements is true about the two functions $f_1(n) = (\log_2(\log_2 n))^2$ and $f_2(n) = (\log_2 n)/10$?

   (a) $f_1(n)$ grows asymptocially faster than $f_2(n)$

   (b) $f_1(n)$ and $f_2(n)$ asymptotically grow at the same rate

   (c) $f_2(n)$ grows asymptotically faster than $f_1(n)$

   Answer (c). Let $m = \log_2 n$. Then, we're comparing $(\log_2 m)^2$ and $m/10$ and we know that $m/10$ (a polynomial in m) grows faster than $(\log_2 m)^2$ (a power of a logarithm).

3. The function $4^{\log_2 n}$ can be simplified to a

   (a) logarithmic function.

   (b) quadratic function.

   (c) linear function.

   (d) sublinear function.

   Answer (b). This algebraic manipulation does the trick: $4^{\log_2 n} = 2^{2\log_2 n} = 2^{\log_2(n^2)} = n^2$.

4. Which of the following statements is true about the two functions $f_1(n) = n^2 \cdot (\log_3 n)^5$ and $f_2(n) = n^3$?

   (a) $f_1(n)$ grows asymptocially faster than $f_2(n)$

   (b) $f_1(n)$ and $f_2(n)$ asymptotically grow at the same rate

   (c) $f_2(n)$ grows asymptotically faster than $f_1(n)$

   Answer (c). Here we are effectively comparing $(\log_3 n)^5$ versus n and we know that the latter grows asymptotically faster.

5. The function $2^{\sqrt{\log_2 n}}$ is a

   (a) logarithmic function.

   (b) polynomial function.

   (c) exponential function.

   (d) sublinear function.

   Answer (d). Notice that $(2^{\sqrt{\log_2 n}})^{\sqrt{\log_2 n}} = 2^{\log_2 n} = n$. Hence, $2^{\sqrt{\log_2 n}}$ grows more slowly than the linear function.

CS:3330 Quiz 1, Spring 2017 (Version 2)

1. Which of the following statements is true about the two functions $f_1(n) = n^{\log_2 16}$ and $f_2(n) = n^3$?

   (a) $f_1(n)$ grows asymptocially faster than $f_2(n)$

   (b) $f_1(n)$ and $f_2(n)$ asymptotically grow at the same rate

   (c) $f_2(n)$ grows asymptotically faster than $f_1(n)$

   Answer (a). $\log_2 16 = 4$

2. Which of the following statements is true about the two functions $f_1(n) = 20 \log_2 n$ and $f_2(n) = 40 \log_4 n$?

   (a) $f_1(n)$ grows asymptocially faster than $f_2(n)$

   (b) $f_1(n)$ and $f_2(n)$ asymptotically grow at the same rate

   (c) $f_2(n)$ grows asymptotically faster than $f_1(n)$

   Answer (b). In this case we use to change of base formula to show something stronger, namely, $f_1 = f_2$.

3. Which of the following statements is true about the two functions $f_1(n) = 2^{(\log_2 n)^2}$ and $f_2(n) = n^{20}$?

   (a) $f_1(n)$ grows asymptocially faster than $f_2(n)$

   (b) $f_1(n)$ and $f_2(n)$ asymptotically grow at the same rate

   (c) $f_2(n)$ grows asymptotically faster than $f_1(n)$

   Answer (a). $2^{(\log_2 n)^2} = (2^{\log_2 n})^{\log_2 n} = n^{\log_2 n}$ and for n large enough, $\log_2 n$ will exceed 20.

4. Which of the following statements is true about the two functions $f_1(n) = n^{\log_n 5}$ and $f_2(n) = n^5$?

   (a) $f_1(n)$ grows asymptocially faster than $f_2(n)$

   (b) $f_1(n)$ and $f_2(n)$ asymptotically grow at the same rate

   (c) $f_2(n)$ grows asymptotically faster than $f_1(n)$

   Answer (c). $n^{\log_n 5} = 5$

5. The function $\sqrt{n} \cdot \log_2 n$ is a

   (a) logarithmic function.

   (b) polynomial function.

   (c) exponential function.

   (d) sublinear function.

   Answer (d). We know that $\log_2 n$ grows slower than any function of the form n^c where $c > 0$. This means that $\sqrt{n} \cdot \log_2 n$ grows slower than say $n^{0.51}$ and is therefore sublinear.

CS:3330 Quiz 1, Spring 2017 (Version 3)

1. Which of the following statements is true about the two functions $f_1(n) = n^{\log_2 n}$ and $f_2(n) = 2^n$?

   (a) $f_1(n)$ grows asymptocially faster than $f_2(n)$

   (b) $f_1(n)$ and $f_2(n)$ asymptotically grow at the same rate

   (c) $f_2(n)$ grows asymptotically faster than $f_1(n)$

   Answer (c). The function $n^{\log_2 n} = (2^{\log_2 n})^{\log_2 n} = 2^{(\log_2 n)^2}$. Now that the bases are the same, we just compare the growth of the exponents.

2. Which of the following statements is true about the two functions $f_1(n) = n^{\log_n 5}$ and $f_2(n) = n^5$?

   (a) $f_1(n)$ grows asymptocially faster than $f_2(n)$

   (b) $f_1(n)$ and $f_2(n)$ asymptotically grow at the same rate

   (c) $f_2(n)$ grows asymptotically faster than $f_1(n)$

   Answer (c). $n^{\log_n 5} = 5$

3. Which of the following statements is true about the two functions $f_1(n) = (1.001)^n$ and $f_2(n) = (0.999)^n$?

   (a) $f_1(n)$ grows asymptocially faster than $f_2(n)$

   (b) $f_1(n)$ and $f_2(n)$ asymptotically grow at the same rate

   (c) $f_2(n)$ grows asymptotically faster than $f_1(n)$

   Answer (a). The growth of these two functions is vastly different. As n gets large, $f_1(n) \to \infty$ and $f_2(n) \to 0$.

4. Which of the following statements is true about the two functions $f_1(n) = 100n^4 + 365n \cdot \log_2 n$ and $f_2(n) = n^4 \cdot \log_2 n - 2000n^4$?

   (a) $f_1(n)$ grows asymptocially faster than $f_2(n)$

   (b) $f_1(n)$ and $f_2(n)$ asymptotically grow at the same rate

   (c) $f_2(n)$ grows asymptotically faster than $f_1(n)$

   Answer (c). Since these functions are a mixture of polynomials and logarithms, we will focus on the largest degree polynomial term. The "coefficient" of n^4 in $f_1(n)$ is 100 and in $f_2(n)$ is $\log_2 n - 2000$. The latter grows with n whereas the former is a constant. So $f_2(n)$ will grow faster with n overall.

5. Which of the following statements is true about the two functions $f_1(n) = 60n^2 + 5n + 3$ and $f_2(n) = 60n^2 + 10n + 30$?

   (a) $f_1(n)$ grows asymptocially faster than $f_2(n)$

   (b) $f_1(n)$ and $f_2(n)$ asymptotically grow at the same rate

   (c) $f_2(n)$ grows asymptotically faster than $f_1(n)$

   Answer (b). Since we are looking at asymptotic growth of two polynomials, we don't consider constants and lower order terms as they do not grow as fast as the term with largest degree.

CS:3330 Quiz 1, Spring 2017 (Version 4)

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1. Which of the following statements is true about the two functions $f_1(n) = 2^{\log_2(\log_2 n)}$ and $f_2(n) = \log_2 n$?
   (a) $f_1(n)$ grows asymptocially faster than $f_2(n)$
   (b) $f_1(n)$ and $f_2(n)$ asymptotically grow at the same rate
   (c) $f_2(n)$ grows asymptotically faster than $f_1(n)$

   Answer (b). In this case, $f_1 = f_2$ which is a stronger statement than (b).

2. The function $2^{\sqrt{\log_2 n}}$ is a
   (a) logarithmic function.
   (b) polynomial function.
   (c) exponential function.
   (d) sublinear function.

   Answer (d). Notice that $(2^{\sqrt{\log_2 n}})^{\sqrt{\log_2 n}} = 2^{\log_2 n} = n$. Hence, $2^{\sqrt{\log_2 n}}$ grows more slowly than the linear function.

3. The function $4^{\log_2 n}$ can be simplified to a
   (a) logarithmic function.
   (b) quadratic function.
   (c) linear function.
   (d) sublinear function.

   Answer (b). This algebraic manipulation does the trick: $4^{\log_2 n} = 2^{2\log_2 n} = 2^{\log_2(n^2)} = n^2$.

4. Which of the following statements is true about the two functions $f_1(n) = n^{\log_2 16}$ and $f_2(n) = n^3$?
   (a) $f_1(n)$ grows asymptocially faster than $f_2(n)$
   (b) $f_1(n)$ and $f_2(n)$ asymptotically grow at the same rate
   (c) $f_2(n)$ grows asymptotically faster than $f_1(n)$

   Answer (a). $\log_2 16 = 4$.

5. The function $4^{3\log_2(\log_2 n)}$ can be simplified to a
   (a) logarithmic function.
   (b) polynomial function.
   (c) exponential function.
   (d) function that is a power of a logarithmic function.

   Answer (d). $4^{3\log_2(\log_2 n)} = 2^{6\log_2(\log_2 n)} = 2^{\log_2(\log_2 n)^6} = (\log_2 n)^6$.
