Midterm Exam
Due in class on October 16th

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1. Here are some questions on Luby's algorithm and its variants.

   (i) Consider Luby's randomized distributed $(\Delta + 1)$-coloring algorithm (as described in class and in Luby's 1993 JCSS paper). Recall that for this algorithm we showed that for any vertex v that is uncolored at the beginning of a phase, the probability that v gets colored in the phase is at least $1/4$. Starting with this fact show that the algorithm runs in $O(\log n)$ rounds with high probability.

   (ii) Consider Luby's randomized distributed MIS algorithm (in Section 8.4 in Peleg's book). What could go wrong if we selected probabilities as $p(v) \leftarrow 1/D_H(v)$ (instead of $p(v) \leftarrow 1/(D_H(v) + 1)$)?

2. Let (V, d) be a metric space. For any α, $0 < \alpha \leq 1$, define an *α-quasi unit ball graph* (in short α-QUBG) as graph with vertex set V and edge set E satisfying the following property: for any $u, v \in V$, $u \neq v$, if $d(u, v) \leq \alpha$ then $\{u, v\} \in E$; also if $d(u, v) > 1$ then $\{u, v\} \notin E$.

   We know that for a UBG in a metric space of constant doubling dimension, the Kuhn-Moscibroda-Wattenhofer algorithm (PODC 2005) computes an MIS in $O(\log^* n)$ rounds. This problem asks you to show that using similar ideas, one can compute an MIS on an α-QUBG in $O(\log^* n)$ rounds as well.

   Suppose we are given an α-QUBG $G = (V, E)$ residing in a metric space (V, d). Further suppose that (V, d) has constant doubling dimension. Also assume that α is a fixed constant.

   (i) State the algorithm. This should be similar to the algorithm described in class (called Algorithm 1 in the paper).

   (ii) Prove that the algorithm computes an MIS.

   (iii) Show that if implemented as described, each iteration of the algorithm takes $O(\log^* n)$ rounds. This would correspond to Lemma 5.2 and Corollary 5.3 in the paper.

   (iv) Show that the algorithm can be implemented in $O(\log^* n)$ rounds. This corresponds to Lemma 5.5.
