1. Here are some questions on Luby’s algorithm and its variants.

(i) Consider Luby’s randomized distributed \((\Delta + 1)\)-coloring algorithm (as described in class and in Luby’s 1993 JCSS paper). Recall that for this algorithm we showed that for any vertex \(v\) that is uncolored at the beginning of a phase, the probability that \(v\) gets colored in the phase is at least \(1/4\). Starting with this fact show that the algorithm runs in \(O(\log n)\) rounds with high probability.

(ii) Consider Luby’s randomized distributed MIS algorithm (in Section 8.4 in Peleg’s book). What could go wrong if we selected probabilities as \(p(v) = 1/D_H(v)\) (instead of \(p(v) = 1/(D_H(v) + 1)\))?  

2. Let \((V,d)\) be a metric space. For any \(\alpha, 0 < \alpha \leq 1\), define an \(\alpha\)-quasi unit ball graph (in short \(\alpha\)-QUBG) as graph with vertex set \(V\) and edge set \(E\) satisfying the following property: for any \(u, v \in V, u \neq v\), if \(d(u, v) \leq \alpha\) then \(\{u, v\} \in E\); also if \(d(u, v) > 1\) then \(\{u, v\} \notin E\).

We know that for a UBG in a metric space of constant doubling dimension, the Kuhn-Moscibroda-Wattenhofer algorithm (PODC 2005) computes an MIS in \(O(\log^* n)\) rounds. This problem asks you to show that using similar ideas, one can compute an MIS on an \(\alpha\)-QUBG in \(O(\log^* n)\) rounds as well.

Suppose we are given an \(\alpha\)-QUBG \(G = (V,E)\) residing in a metric space \((V,d)\). Further suppose that \((V,d)\) has constant doubling dimension. Also assume that \(\alpha\) is a fixed constant.

(i) State the algorithm. This should be similar to the algorithm described in class (called Algorithm 1 in the paper).

(ii) Prove that the algorithm computes an MIS.

(iii) Show that if implemented as described, each iteration of the algorithm takes \(O(\log^* n)\) rounds. This would correspond to Lemma 5.2 and Corollary 5.3 in the paper.

(iv) Show that the algorithm can be implemented in \(O(\log^* n)\) rounds. This corresponds to Lemma 5.5.