QuickSort: Final Lecture

MAY 2ND, 2014

The partition function

```
def partition(L, first, last):
    # We pick the element L[first] as the "pivot" around which we partition the list
    p = first

    # We process the rest of the elements, one-by-one, in left-to-right order
    for current in range(p+1, last+1):
        # If L[current] is smaller than the pivot, it needs to move into the first block,
        # to the left of the pivot.
        if L[current] < L[p]:
            swap(L, current, p+1)
            swap(L, p, p+1)
            p = p + 1

    return p
```

The partition function in action

- Suppose

```
L = [7, 2, 13, 19, 3, 19, 8, 11, 12, 16, 1, 7]
```

- Say we call

```
partition(L, 0, 11)
```

First few iterations of partition

- Processed Unprocessed

[ ] 7 [ ] || [2, 13, 19, 3, 19, 8, 11, 12, 16, 1, 7]

swaps: 2 ↔ 2, 2 ↔ 7

[2] 7 [ ] || [13, 19, 3, 19, 8, 11, 12, 16, 1, 7]

0 swaps

[2] 7 [13] || [19, 3, 19, 8, 11, 12, 16, 1, 7]

0 swaps

[2] 7 [13, 19] || [3, 19, 8, 11, 12, 16, 1, 7]

swaps: 3 ↔ 13, 3 ↔ 7

[2, 3] 7 [19, 13] || [19, 8, 11, 12, 16, 1, 7]

The rest of the iterations

[2, 3] 7 [19, 13] || [19, 8, 11, 12, 16, 1, 7]
[2, 3] 7 [19, 13, 19] || [8, 11, 12, 16, 1, 7]
[2, 3] 7 [19, 13, 19, 8] || [11, 12, 16, 1, 7]
[2, 3] 7 [19, 13, 19, 8, 11] || [12, 16, 1, 7]
[2, 3] 7 [19, 13, 19, 8, 11, 12] || [16, 1, 7]
[2, 3] 7 [19, 13, 19, 8, 11, 12, 16] || [1, 7]
[2, 3, 1] 7 [13, 19, 8, 11, 12, 16, 19] || [7]
[2, 3, 1] 7 [13, 19, 8, 11, 12, 16, 19, 7] ||

The function returns 3.

The QuickSort function

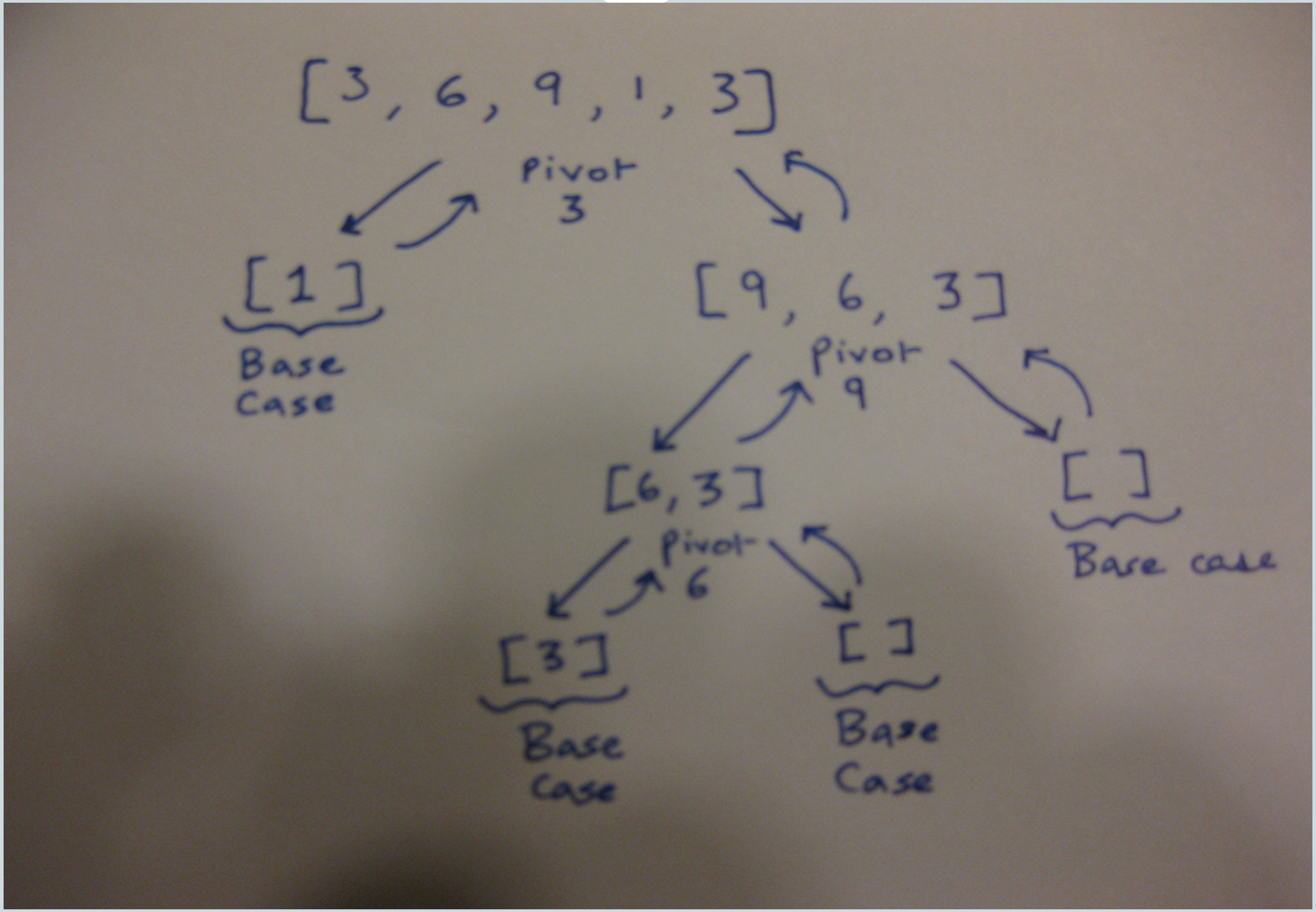
```
def generalQuickSort(L, first, last):
    # Base case: if first == last, then there is only one element in the
    # slice that needs sorting. So there is nothing to do.

    # Recursive case: if there are 2 or more elements in the slice L[first:last+1]
    if first < last:
        # Divide step: partition returns an index p such that
        # first <= p <= last and everthing in L[first:p] is <= L[p]
        # and everything in L[p+1:last+1] is >= L[p]
        p = partition(L, first, last)

        # Conquer step
        generalQuickSort(L, first, p-1)
        generalQuickSort(L, p+1, last)

        # Combine step: there is nothing left to do!
```

quickSort in action

- L = [3, 6, 9, 1, 3]. Suppose we call quickSort(L).

```
Calling quicksort on  [3, 6, 9, 1, 3]
Divide step gives  [1] 3 [9, 6, 3]
Calling quickSort on  [1]
Calling quickSort on  [9, 6, 3]
Divide step gives  [6, 3] 9 []
Calling quickSort on  [6, 3]
Divide step gives  [3] 6 []
Calling quickSort on  [3]
Calling quickSort on  []
Calling quickSort on  []
```

quickSort in action

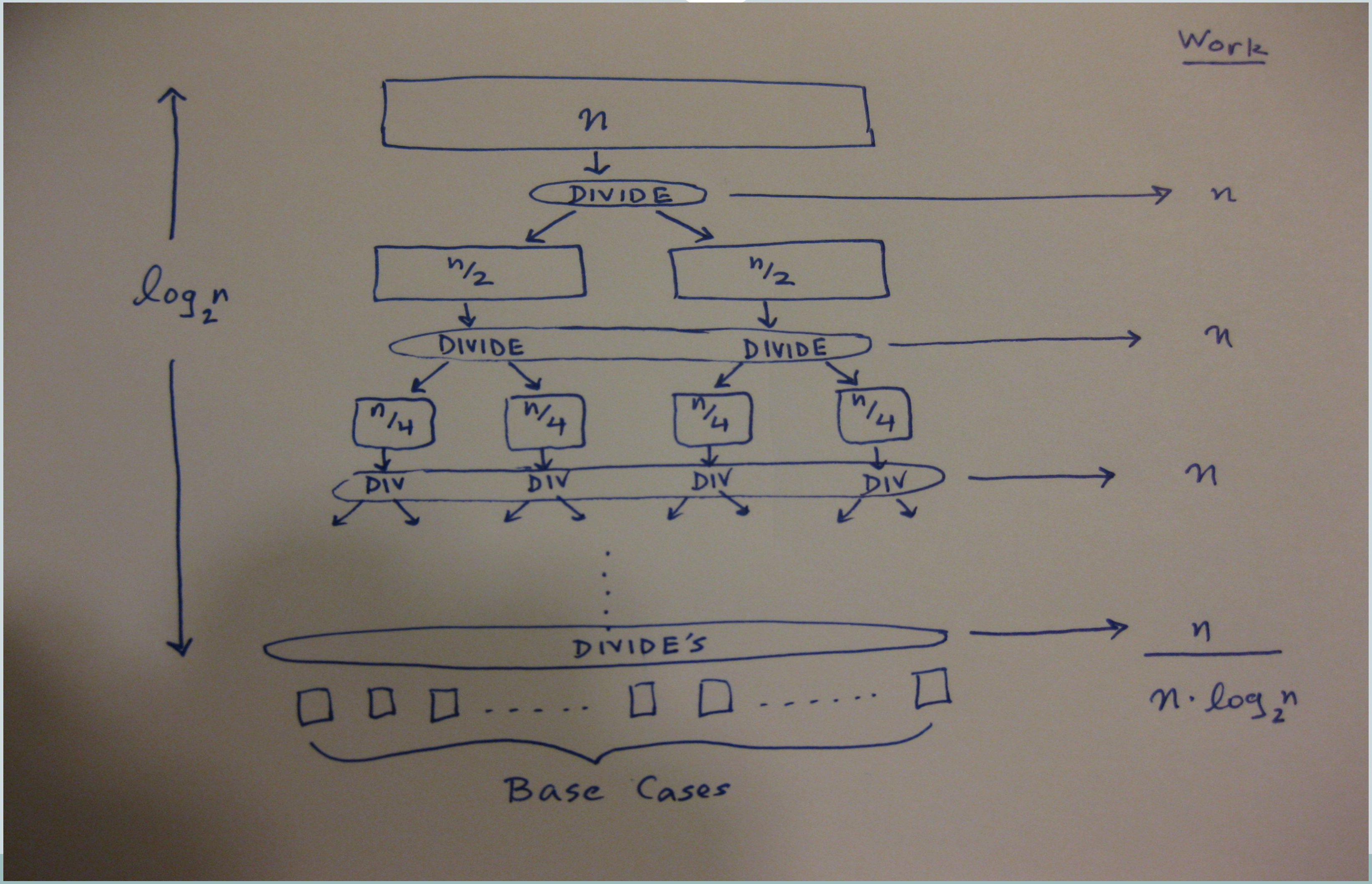


Efficiency of quickSort

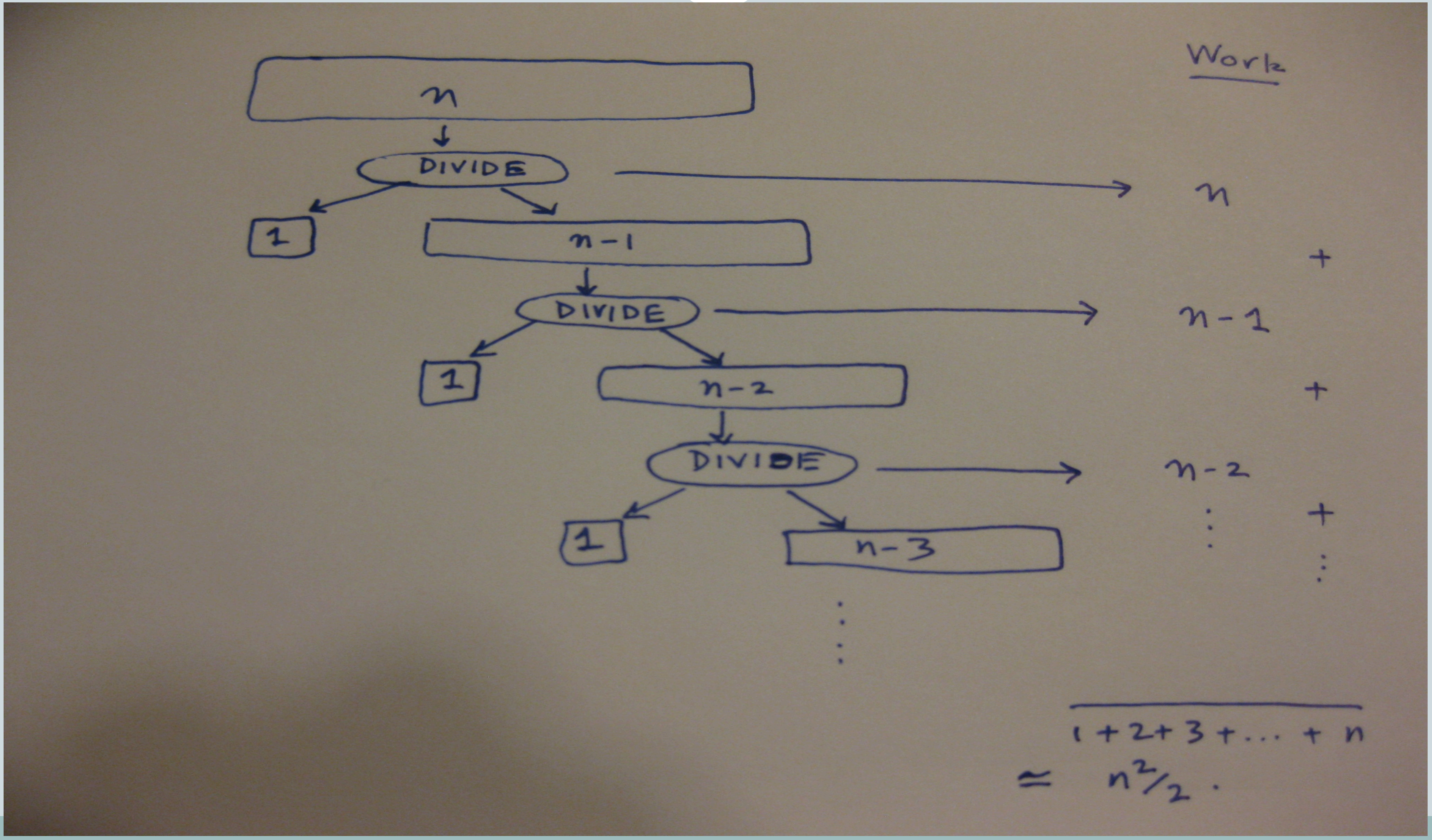
- **Key observation 1**: partition was designed so as to take n steps on a list of size-n.

- **Key observation 2:** the relative sizes of the two blocks resulting from partition plays a critical role in determining the overall running time of quickSort.

Best case example



Worst case example



So how does one pick a good pivot?

Simple (and effective) solution:

Pick a random element as the pivot!

Code

```
# Execute these two lines of code at the
# beginning of partition
r = random.randint(first, last)
swap(L, first, last)
```