Our First Programming Problem

JAN 24TH 2014
Problem: Converting decimal numbers to binary

- Given a non-negative integer, convert it into its binary equivalent.

Example:
- Input: 123 Output: 1111011
- Input: 1363 Output: 10101010011
- Input: 12 Output: 1100
Plan of Action

1. Understand the problem. What does “binary equivalent” mean?

2. Design an algorithm for the problem. How would we solve the problem with a pencil and paper?

3. Write down pseudocode for the algorithm.

4. Translate the pseudocode into Python code.

5. Think about correctness and test.

6. Think about efficiency. Is the algorithm too slow?
This example will illustrate...

- Constants
- Variables
- Operators
- Data types
- Expressions
- Function calls
- Input statements
- Output statements
- Control flow statements
Decimal numbers revisited

Consider the decimal number 8,374.

<table>
<thead>
<tr>
<th>Digits</th>
<th>8</th>
<th>3</th>
<th>7</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Place value</td>
<td>1000</td>
<td>100</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

Therefore, the “value” of this number is

$$8 \times 1000 + 3 \times 100 + 7 \times 10 + 4 \times 1$$
What are binary numbers?

Similarly, consider the binary number 10110110.

<table>
<thead>
<tr>
<th>Bits:</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Place values:</td>
<td>128</td>
<td>64</td>
<td>32</td>
<td>16</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Just like the place values for decimal numbers are powers of 10, the place values for *binary numbers* are powers of 2.

Therefore, the “value” of this number is

\[ 128 + 32 + 16 + 4 + 2 = 182 \]
## Table of Binary Equivalents

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Binary</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>101</td>
</tr>
<tr>
<td>6</td>
<td>110</td>
</tr>
<tr>
<td>7</td>
<td>111</td>
</tr>
<tr>
<td>8</td>
<td>1000</td>
</tr>
<tr>
<td>9</td>
<td>1001</td>
</tr>
<tr>
<td>10</td>
<td>1010</td>
</tr>
<tr>
<td>11</td>
<td>1011</td>
</tr>
<tr>
<td>12</td>
<td>1100</td>
</tr>
<tr>
<td>13</td>
<td>1101</td>
</tr>
<tr>
<td>14</td>
<td>1110</td>
</tr>
</tbody>
</table>
Two observations based on this table

**Observation 1:**
If n is even, then its binary equivalent ends with a 0; otherwise, if n is odd, its binary equivalent ends with 1.

(Can you prove this?)
Observation 2:
Suppose that the binary equivalent of n is
\[ b_k \ldots b_2 b_1 b_0. \]
If n is even, then the binary equivalent of n/2 is
\[ b_k \ldots b_2 b_1 \]
and if n is odd, then the binary equivalent of (n-1)/2 is
\[ b_k \ldots b_2 b_1. \]

(Can you prove this?)
This suggests an algorithm

1. Check if the given number $n$ is odd or even.

2. If $n$ is even, we know that its binary equivalent ends with 0. Furthermore, to get the rest of $n$’s binary equivalent, we need to “process” $n/2$.

3. If $n$ is odd, we know that the binary equivalent ends with 1. Furthermore, to get the rest of $n$’s binary equivalent, we need to “process” $(n-1)/2$. 
What is an algorithm?

- An algorithm is a step-by-step procedure to complete a task.

**Examples of algorithms:**
- A recipe for baking muffins,
- The output produced by Google maps when you ask for directions from Iowa City to Santa Fe,
- The procedure for computing the binary equivalent of a decimal integer described in the previous slide.

- The oldest example of a computational algorithm: the 2300-year old *Euclid’s algorithm* for computing the greatest common divisor.

- Your digital life depends on algorithms: web search algorithms, cryptography algorithms, data compression algorithms, etc.
Illustration of our algorithm

Let the given input be $n = 203$.

1. $n = 203$ is odd. So rightmost bit is 1. 
   To get the rest of the answer we should “process” $(n-1)/2 = 101$.
2. $n = 101$ is odd. So the rightmost bit is 1. 
   To get the rest of the answer we should “process” $(n-1)/2 = 50$
3. $n = 50$ is even. So the rightmost bit is 0. 
   To get the rest of the answer we should “process” $n/2 = 25$.
4. $n = 25$ is odd. So the rightmost bit is 1. 
   To get the rest of the answer we should “process” $(n-1)/2 = 12$.
5. $n = 12$ is even. So the rightmost bit is 0. 
   To get the rest of the answer we should “process” $n/2 = 6$.
6. $n = 6$ is even. So the rightmost bit is 0. 
   To get the rest of the answer we should “process” $n/2 = 3$.
7. $n = 3$ is odd. So the rightmost bit is 1. 
   To get the rest of the answer we should “process” $(n-1)/2 = 1$.
8. $n = 1$ is odd. So the rightmost bit is 1. 
   To get the rest of the answer we should “process” $(n-1)/2 = 0$.

So the output (right to left) is 1 1 0 1 0 0 1 1.
Pseudocode

1. Read the number n given as input.
2. If n is even, output 0. Replace n by n/2.
3. If n is odd, output 1. Replace n by (n-1)/2.
4. If n is 0, stop. Otherwise go to Line 2.

Note that this algorithm produces the binary equivalent of n in “right to left order.”
What is pseudocode?

- Pseudocode is a “language” used to describe algorithms.

- It is not as precise as actual programming language code.

- But it is precise enough that we can reason about correctness and efficiency of the algorithm.
n = int(raw_input("Enter a positive integer:"))
while n > 0:
    print n % 2
    n = n/2