1. (Problem 5.12) Let $E = \{ <M> | M \text{ is a single-tape TM which ever writes a blank symbol over a non-blank symbol on any input} \}$. We show that $A_{TM}$ reduces to $E$. Assume for the sake of contradiction that $E$ is decidable, and let $R$ be a TM that decides $E$. We can use $R$ to construct a TM $S$ that decides $A_{TM}$. The TM $S$ works as follows:

**TM $S$:** On input $<M, w>$
1. Use $M$ and $w$ to construct TM $T_w$.
   **TM $T_w$:** On any input
   a. Simulate $M$ on $w$. Use symbol $\sqcup'$ instead of $\sqcup$ when writing and treat it like $\sqcup$ when reading.
   b. If $M$ accepts, write a true blank symbol.
2. Run $R$ on $<T_w>$ to determine whether $T_w$ ever writes a blank.
3. If $R$ accepts, $M$ accepts $w$, therefore accept. Otherwise reject.

2. (Problem 5.13) Use the universal turing machine described in the textbook.

3. (Problem 5.14) Let $L_{TM} = \{ <M, w> | M \text{ on } w \text{ tries moving it’s head left from the leftmost cell, at some point in it’s computation} \}$. Assume to the contrary that TM $R$ decides $L_{TM}$. Construct a TM $S$ that uses $R$ to decide $A_{TM}$.

**TM $S$:** On input $<M, w>$
1. Convert $M$ to $M'$, where $M'$ first moves it’s input over one square to the right, and writes a new symbol $\$$ on the leftmost tape cell. Then $M'$ simulates $M$ on the input.
If $M'$ ever sees a $\$, then $M'$ moves its head one square to the right and remains in the same state. If $M$ accepts, $M'$ moves its head all the way to the left and then moves left off the leftmost tape cell.

2. Run $R$ on $< M', w >$.

3. If $R$ accepts, accept. If $R$ rejects, reject.

4. (Problem 5.15) Consider the length of the shortest computation path of a TM that would result in a left move. You can use this to design a TM that decides the language.

5. (Problem 5.17) Try to find the conditions under which dominos with a unary alphabet can form a match. Use this observation to design a TM that decides PCP over a unary alphabet.

6. (Problem 5.19) Any match for SPCP starts with a domino that has two equal strings, and therefore is a match all by itself. So we only need to check whether the input contains a domino that has two equal strings. If so, accept, else reject.

7. (Problem 5.33) We need to show two reductions. The first to show $S$ is not turing recognizable, reduce $A_{TM}$ to $\overline{S}$, and similarly to show $\overline{S}$ is not turing recognizable, reduce $A_{TM}$ to $S$.

8. (Problem 5.35) Design a TM that decides $X$... or show that $X$ is not decidable by reducing any undecidable problem to $X$. 

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