Chapter 1 Section 3. Solution Sets

Suppose we consider a matrix

$$\mathbf{AA} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix};$$

The corresponding homogeneous equations using variables x y and z are:

$$\mathbf{AA}.\begin{pmatrix} x \\ y \\ z \end{pmatrix} == \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\{\{x+y+z\},\{x+z\},\{2x+y+2z\}\} == \{\{0\},\{0\},\{0\}\}$$

$$\mathbf{Solve}\left[\mathbf{AA}.\begin{pmatrix} x \\ y \\ z \end{pmatrix} == \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \{x,y,z\}\right]$$

$$\{\{x \to -z, y \to 0\}\}$$

we can see this be looking at the row reduced form of AA

RowReduce[AA]//MatrixForm

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In our text we would express this with parameter t. So we let z=t and solution set is all vectors:

$$\begin{pmatrix} -t \\ 0 \\ t \end{pmatrix};$$

or equivalently

$$\boldsymbol{t}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix};$$

Next we look at non homogeneous equations with same matrix AA :

$$\textbf{AA.}\begin{pmatrix} x \\ y \\ z \end{pmatrix} == \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\{\{x+y+z\},\{x+z\},\{2x+y+2z\}\} == \{\{1\},\{2\},\{3\}\}$$

$$\textbf{Solve}\left[\textbf{AA.}\begin{pmatrix} x \\ y \\ z \end{pmatrix} == \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \{x, y, z\}\right]$$

$$\{\{x \to 2 - z, y \to -1\}\}$$

We see there are infinitely many solutions. We can see this also from row reduction.

$$\textbf{RowReduce}\left[\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 2 & 3 \end{pmatrix}\right] \textbf{//MatrixForm}$$

$$\begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Here is another set of equations using AA. This time there are no solutions

$$\textbf{AA.}\begin{pmatrix} x \\ y \\ z \end{pmatrix} == \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$\{\{x+y+z\},\{x+z\},\{2x+y+2z\}\} == \{\{3\},\{2\},\{1\}\}$$

$$\textbf{Solve}\left[\text{AA}.\begin{pmatrix} x \\ y \\ z \end{pmatrix} == \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \{x, y, z\}\right]$$

$\{\}$

$$\textbf{RowReduce}\left[\begin{pmatrix} 1 & 1 & 1 & 3 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 2 & 1 \end{pmatrix}\right] \textbf{//MatrixForm}$$

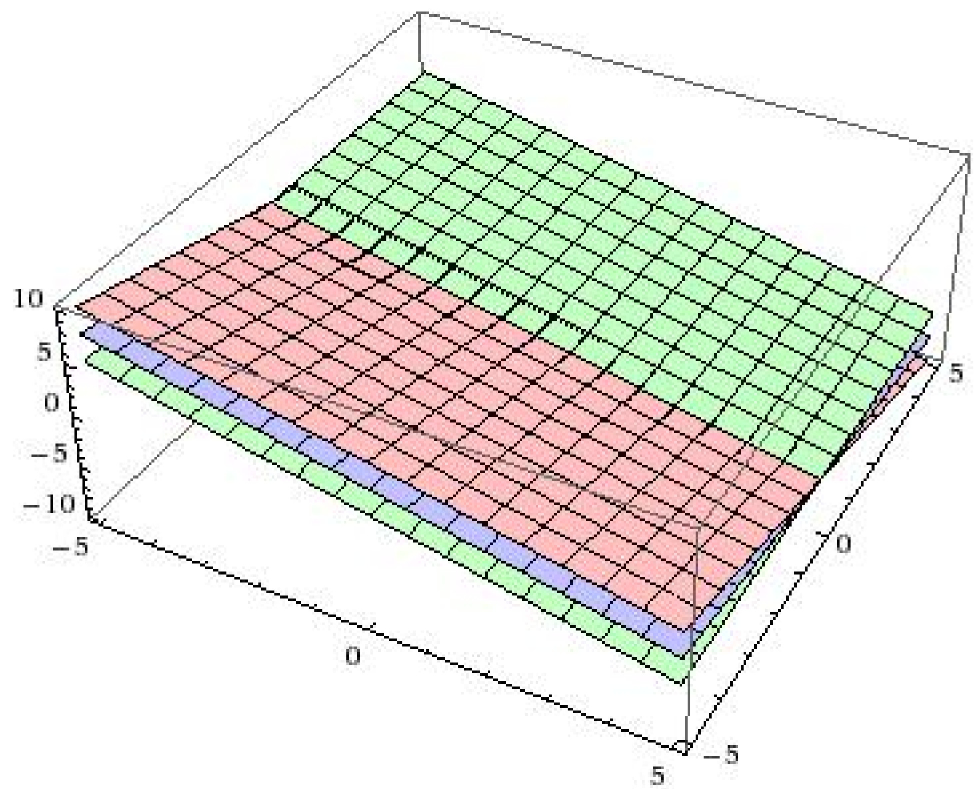
$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

We can understand this geometrically by looking at plots of these equations.

NOTE : to plot an equations, I solved for z and plotted as function of x and y

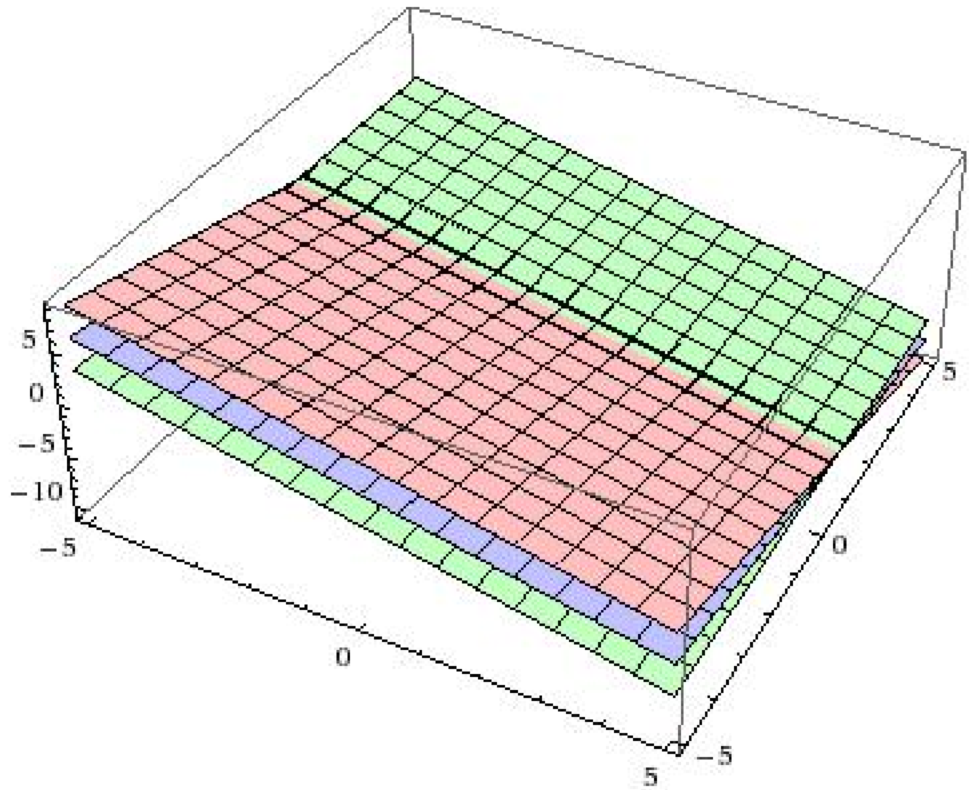
The solution set of the first set of equations is a line which contains the origin

$$\textbf{Plot3D}[\{-x - y, -x, -x - y/2\}, \{x, -5, 5\}, \{y, -5, 5\}]$$



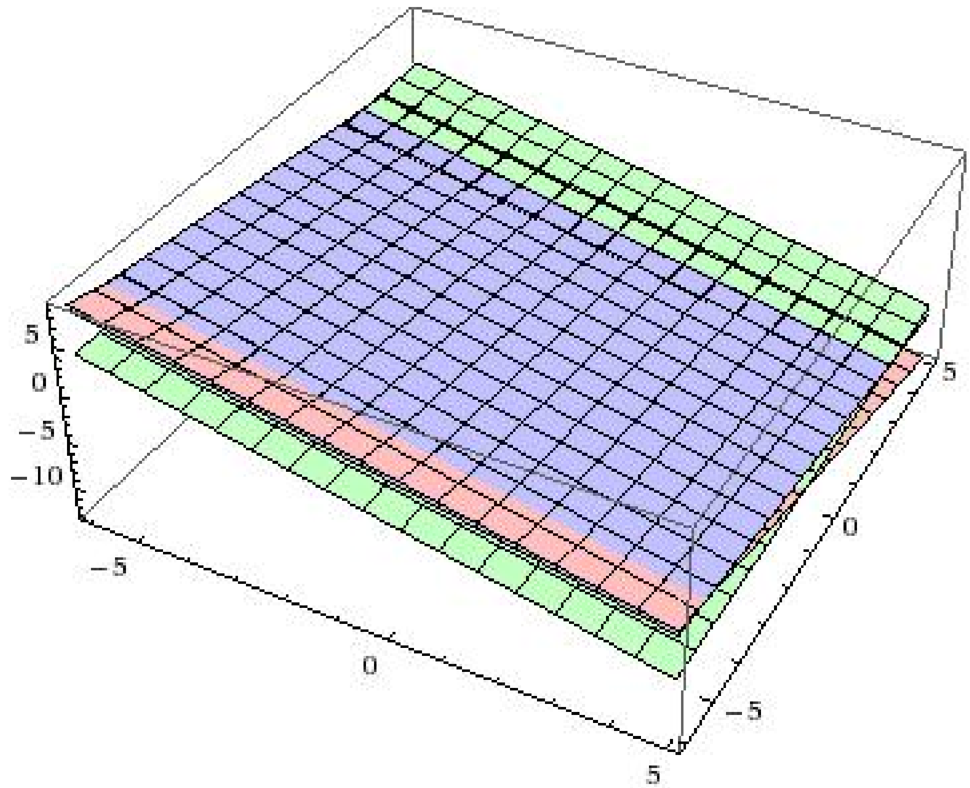
The second set has solution set which is a line parallel to the homogeneous set

Plot3D$[\{-x-y-1, -x-2, -x-(y+3)/2\}, \{x, -5, 5\}, \{y, -5, 5\}]$



However the third set has no solutions at all. The three lines of intersection of pairs of planes give three parallel lines.

Plot3D[{$-x-y-3, -x-2, -x-(y+1)/2$}, {$x, -5, 5$}, {$y, -5, 5$}]


5
0
−5
−10
−5
0
5
5
0
−5