$22\mathbf{C} : 231$ ($CS : 5350 : 0001$) **Design and Analysis of Algorithms**
Homework 6: Poly-time Reducibility and NP-completeness

The homework has two types of problems – reinforcement problems and regular problems. For the reinforcement problems, we are not concerned with originality in coming up with the solution, but rather with how well you write up the solution. You can get help in coming up with the solution – from friends, online, etc. – but understand the solution and explain it in your own words. For the regular problems, the only type of help you can get is collaboration with classmates, and discussion with the instructor or TA. No record or notes, electronic or written, should be taken from such collaborations. For these problems we do care about originality in coming up with the solution.

The homework is worth 10 points (each problem is worth 2.5), and is due in class on Thursday, May 9. **For the purpose of mastering the material, it is highly recommended that you make a serious effort to solve even the reinforcement problems by yourself.**

Regular Problems

1. In the 3-colorability problem (which we study in class on May 2), we are given an undirected graph $G = (V, E)$, and we want to determine if there is a function (coloring) $\chi : V \to \{1, 2, 3\}$ such that $\chi(u) \neq \chi(v)$ for each edge $(u, v) \in E$. Give a direct polynomial time reduction from this problem to the CNF satisfiability problem. By a direct reduction, we mean one that does not use the known fact that CNF satisfiability is NP-complete. **Hint:** Have a boolean variable x_{ij} for each $i \in V$ and $j \in \{1, 2, 3\}$, with the interpretation that $x_{ij} = 1$ if $\chi(i) = j$.

2. A long expedition to Antarctica is being planned, and there are n roles for which candidates need to be chosen. (Examples of roles might be marine expert, entertainer, and so on.) For the i'th role, there is a list L_i of candidates, and exactly one of these candidates must be chosen for the i'th role. (No person is a candidate on more than one list.) The only catch is that there is a set P of pairs of people that simply don't get along. So if $(X, Y) \in P$, then both X and Y cannot be chosen. The *Expedition Planning* Problem is to determine (given n, the list L_i for each $1 \leq i \leq n$ (such that no person is on more than one list), and the set P of incompatible pairs) if it is possible to choose a candidate for each of the roles so that for any $(X, Y) \in P$, both X and Y are not chosen.

   For example, suppose $n = 3$, and $L_1 = \{A, B\}, L_2 = \{C, D\}, L_3 = \{E\}$, and $P = \{(A, C), (B, D), (A, E), (C, E)\}$. Then such a choice is not possible. On the other hand, if $L_1 = \{A, B, F\}$, such a choice is possible. We can pick F, D, and E.

   Show that *Expedition Planning* is NP-complete.

Reinforcement Problems

1. Exercise 1 of Chapter 8.

2. Exercise 4 of Chapter 8.