22C : 231 ($CS : 5350 : 0001$) Design and Analysis of Algorithms
Homework 2

The homework has two types of problems – reinforcement problems and regular problems. For the reinforcement problems, we are not concerned with originality in coming up with the solution, but rather with how well you write up the solution. You can get help in coming up with the solution – from friends, online, etc. – but understand the solution and explain it in your own words. For the regular problems, the only type of help you can get is collaboration with classmates, and discussion with the instructor or TA. No record or notes, electronic or written, should be taken from such collaborations. For these problems we do care about originality in coming up with the solution.

The homework is worth 10 points, with each question being worth 2 points. The theme is dynamic programming and the use of recursion therein. The homework is due in class on Thursday, Feb 21.

Reinforcement Problems

1. Exercise 4 of Chapter 6.

2. Exercise 9 of Chapter 6.

Regular Problems

1. Let us say that point $p = (p_x, p_y)$ is *dominated by* point $q = (q_x, q_y)$ (denoted $p \prec q$) if $p_x \leq q_x$ and $p_y \leq q_y$. We are given a set $P = \{p_1, \ldots, p_n\}$ of n distinct points in the plane. The goal is find a maximum sized sequence $\langle p_{i_1}, p_{i_2}, \ldots, p_{i_k} \rangle$ such that (a) The points in the sequence are distinct, and belong to P, and (b) $p_{i_1} \prec p_{i_2} \prec \cdots \prec p_{i_k}$. Develop a polynomial time algorithm for this problem.

   Hint: You can apply dynamic programming directly, or you can reduce to a shortest path problem in graphs without negative weight cycles, which we'll talk about shortly.

2. We are given a rooted tree where each node has a weight that can be any integer (positive, negative, or zero). The goal is to find a subtree that includes the root and maximizes the sum of the weights of the nodes in it. Develop a polynomial time algorithm for this problem. You can assume that the input tree is represented in any reasonable way. (For example, there is an object corresponding to each node. Such a node object stores its weight and has a reference to its parent as well as to a list of its children. The parent of the root is null. The tree is accessed via a reference to its root.)

3. In a school there are $2n$ classrooms, and for each classroom i, there is a boy representative with integer value $b_i \geq 0$ and a girl representative with integer value $g_i \geq 0$. We want to pick a delegation of $2n$ representatives, one from each classroom, so that

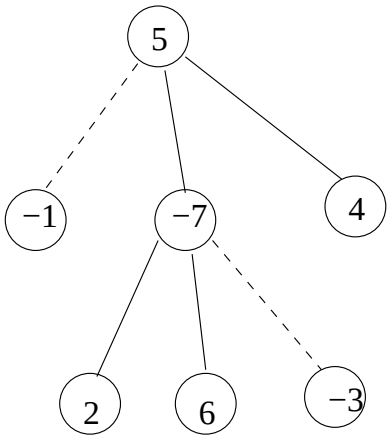

Figure 1: Regular Problem 2: the non-dashed edges identify the optimal tree

the delegation has exactly n boys and n girls. Among such delegations, the goal is to pick one that maximizes the sum of the values of the members of the delagation.

Develop an algorithm for the problem with running time polynomial in n and V, where V is the maximum value of any boy or girl.