22C : 196 Computational Geometry
Homework 1

The problems in this homework are adapted from the text *Computational Geometry: Algorithms and Applications* by de Berg et al., but I have stated the problems to avoid issues that may come up because of using different versions.

1. Let S be a set of n disjoint line segments whose upper endpoints lie on the line $y = 1$ and whose lower endpoints lie on the line $y = 0$. These segments partition the horizontal strip $[-\infty, +\infty] \times [0, 1]$ into $n+1$ regions. Give an $O(n \log n)$ time algorithm that takes as input S and creates a data structure, so that given any query point q in the strip, the region containing q can be determined in $O(\log n)$ time. Describe the query algorithm in some detail. (3 points)

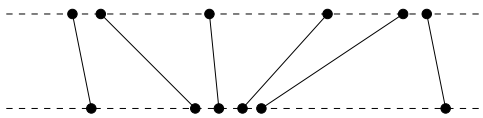

Figure 1: Problem 1

   For giving the answer to a query, you can "label" a region in any way that you deem reasonable. Also make a reasonable choice for what to output for a query point that lies on some segment.

2. Let S be a set of n circles in the plane. Describe a plane sweep algorithm to compute all intersection points between the circles. (Because we deal with circles, not discs, two circles do not intersect if one lies entirely inside the other.) Your algorithm should run in $O((n + k) \log n)$ time, where k is the number of intersection points. (3 points)

   (Make the general position assumption that no three circles intersect at a point.)

3. Let S be a set of n disjoint line segments in the plane, and let p be a point not on any of the line segments of S. We wish to determine all line segments of S that p can see, that is, all line segments of S that contain some point q so that the open segment $\overline{pq}$ doesn't intersect any line segment of S. Give an $O(n \log n)$ time algorithm for this problem that uses a rotating half-line with its endpoint at p. (4 points)

The last two problems can be solved via a modification of the plane-sweep approach. If you are doing this, you should describe the sweep-line status (or its equivalent), the event points, the data structures used for maintaining these, and the actions taken at each type of event point. You should also outline how the desired running time bound follows. I am not looking for great detail, but for the key ideas.

The homework is to be turned in into the dropbox *Homework1* on ICON. I would prefer if you type in the text, but hand-drawn figures are okay. The homework is due by 11:59 pm on Feb 7th.

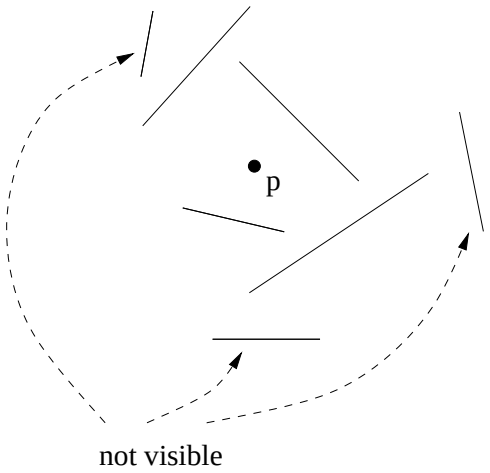

Figure 2: Problem 3

On the question of collaboration and seeking help, I recommend thinking about each problem for 30 minutes first (not counting time spent getting familiar with basic material covered in class). You may collaborate with classmates after that, but definitely avoid looking at completely written solutions of others. Explain the final solution in your own words, and do not turn in a solution that you don't understand.