1. Let the joint p.d.f. of $X$ and $Y$ be $f(x, y) = \frac{xy^2}{30}$ for $x = 1, 2, 3$ and $y = 1, 2$.
   (a) What is the expected value of $X$?
   (b) What is the mean of $Y$?
   (c) What is the covariance of $X$ and $Y$?
   (d) What is the covariance of $X$ and $3Y$?

2. Let $R$, $S$, and $T$ be independent, equally likely events with common probability $\frac{1}{3}$. What is $P[R \cup S \cup T]$?

3. A box contains 4 red balls and 6 white balls. A sample of size 3 is drawn without replacement from the box. What is the probability of obtaining 1 red ball and 2 white balls, given that at least 2 of the balls in the sample are white?

4. When sent a questionnaire, 50% of the recipients respond immediately. Of those who do not respond immediately, 40% respond when sent a follow-up letter. If the questionnaire is sent to 4 persons and a follow-up letter is sent to any of the 4 who do not respond immediately, what is the probability that at least 3 never respond? (Note: this is a difficult problem)

5. If Richard takes his date to a fancy restaurant, 75% of the time, he gets a good-night kiss. During the last month, Richard took seven different dates out to fancy restaurants. You may assume each date is independent.
   (a) What is the probability that exactly 3 of these 7 dates kissed Richard goodnight?
   (b) What is the probability that he was kissed goodnight by at least 5 of his dates?

6. In a large population of people, 50% are married, 20% are divorced, and 30% are single (never married). In a random sample of 5 people, what is the probability that exactly 4 are married?

7. A fair coin is tossed. If a head occurs, 1 fair die is rolled; if a tail occurs, 2 fair dice are rolled. If $Y$ is the total on the die or dice, then find $P[Y = 6]$.

8. What is the probability that a hand of 5 cards chosen randomly and without replacement from a standard deck of 52 cards contains the king of spades, exactly 1 other king, and exactly 2 queens?
9. The number of power surges in an electric grid has a Poisson distribution with a mean of 1 power surge every 12 hours. What is the probability that there will be no more than 2 power surges in a 48-hour period?

10. A pair of dice is tossed 10 times in succession. What is the probability of observing no 7’s and no 11’s in any of the 10 tosses?

11. A card hand selected from a standard card deck consists of 2 kings, a queen, a jack, and a ten. Three additional cards are selected at random and without replacement from the remaining cards in the deck. What is the probability that the enlarged hand contains at least 3 kings?

12. Suppose 30 percent of all electrical fuses manufactured by a certain company fail to meet municipal building standards. What is the probability that in a random sample of 10 fuses, exactly 3 will fail to meet municipal building standards?

13. An urn contains 3 red balls, 2 green balls, and 1 yellow ball. Three balls are selected at random and without replacement from the urn. What is the probability that at least 1 color is NOT drawn?

14. Let the joint p.d.f. of X and Y be defined by \( f(x, y) = \frac{x + y}{32} \) for \( x = 1, 2, y = 1, 2, 3, 4 \).
   (a) What is \( P(X + Y = 3) \)?
   (b) Find \( P(Y = 2X) \)
   (c) Find \( \text{Cov}(X,Y) \).

15. In a certain communications system, there is an average of 1 transmission error per 10 seconds. Let the distribution of transmission errors be Poisson.
   (a) What is the probability of more than 1 error in a communication one-half minute in duration?
   (b) What is the expected number of transmission errors in a 5 minute communication?

16. A bin of 10 lightbulbs contains 4 that are defective. If 3 bulbs are chosen without replacement from the bin, what is the probability that exactly 2 of the bulbs in the sample are defective?

17. A card is drawn at random from an ordinary deck of 52 cards and replaced. This is done a total of 5 independent times. What is the conditional probability of drawing the ace of spades exactly 4 times, given that this ace is drawn at least 4 times?
18. Let $X$ be a random variable such that $E(X^2) = 11$. Suppose $Y$ is a random variable defined by $Y = 4X + b$ where $b$ is unknown. If $Var(Y) = 32$, find $E(X)$.

19. Suppose that $X$ has the probability distribution given by:

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P[X=x]$</td>
<td>.2</td>
<td>.3</td>
<td>.4</td>
<td>$p$</td>
</tr>
</tbody>
</table>

What is the variance of $X$?

20. Sixty percent of new drivers have had driver education. During their first year, new drivers without driver education have probability 0.08 of having an accident, but new drivers with driver education have only a 0.05 probability of an accident.

(a) What is the probability that a new driver chosen at random will have an accident in the first year?

(b) What is the probability a new driver has had driver education, given that the driver has had no accidents the first year?

21. Suppose $X$ has a Poisson distribution with a standard deviation of 4. What is the conditional probability that $X$ is exactly 1 given that $X \geq 1$?