

Models

- Abstraction of reality
- Simplified by ignoring
 - “irrelevant variables”
 - “higher order terms”
 - In Computer Science, we do this with $O(n)$ notation

*You can learn a lot from a model,
but models are not reality!*

Example: Coralville Reservoir

Variables:

- $inflow(t)$ – inflow at time t (volume per unit time)
- $outflow(t)$ – outflow at time t
- $volume(t)$ – reservoir volume at time t
- $level(t)$ – reservoir level at time t
- $gate-setting(t)$ – level of the top of the gate at time t
- $f1(v)$ – level as a function of volume v
- $f2(h)$ – outflow as a function of height-over-gate h

Example: Coralville Reservoir

Rules:

- $\text{volume}(t) = \text{volume}(t - 1) + \text{inflow}(t) - \text{outflow}(t)$
- $\text{level}(t) = f1(\text{volume}(t))$
- $\text{outflow}(t) = f2(\text{level}(t) - \text{gate-setting}(t))$

What simplifying assumptions did we make?

Example: Orbital Dynamics

Variables:

- T – time
- $r(t)$ – distance from earth to satellite, a function of t
- $v(t)$ – velocity of satellite, a function of t
- $a(t)$ – acceleration of satellite, a function of t
- $f(t)$ – force on satellite, a function of t

$r(t)$, $v(t)$, $a(t)$, $f(t)$ *are all vectors with x , y components*

Example: Orbital Dynamics

Rules:

- $f(t) = Gm_e m_s / |r(t)|^2$
- $a(t) = f(t) / m_s$
- $v(t + \Delta t) = v(t) + a(t)\Delta t$
- $r(t + \Delta t) = r(t) + v(t)\Delta t$

What simplifying assumptions did we make?

Example: A Highway Network

Objects:

- Roads
 - travel time
 - source intersection
 - destination intersection
- Intersections
 - rule: priority to the right, stop light, stop sign
- Vehicles
 - current location, a road or an intersection

Example: A Highway Network

- **Events:**
- Vehicle enters road
 - predicts vehicle arrives at destination intersection
 - if a vehicle is waiting, may predict next road entry
- Vehicle arrives at intersection
 - may predict vehicle enters next road
 - vehicle may have to wait its turn
- State of stop light changes
 - if a vehicle is waiting, may predict next road entry

Other Examples

- **Digital Logic:**
 - Gates and wires replace intersections and roads
 - true-false changes instead of vehicles
- **Neuron Networks:**
 - Cell bodies and axons replace intersections and roads
 - action potentials instead of vehicles

How have we simplified? What have we ignored?

Example: Epidemic dynamics

Objects:

- People
 - where do they work, study, shop
 - what household do they live in
 - at each instant, where are they
- Places
 - what are the hours of business
 - at each instant, who is there

Example: Epidemic dynamics

Events:

- Person arrives at a place
 - predicts when they depart depending on place
- Person departs from a place
 - predicts when they arrive at the next place
 - determines infection
 - this depends on length of stay
 - this depends on number of infected people present

What have we ignored? How have we simplified?

Everything is an Object

**In the discrete models just given
we used object-oriented terminology without even
thinking about it.**

This is where the terminology comes from!