1. (30 points) Design an efficient sorting algorithm for an array of $n$ integers where for each element $x$ of the array, $0 < x < n^2$. Please provide the complexity of your algorithm.

2. (30 points) Given a list $L = [a_1, a_2, \ldots, a_n]$ of integers, the longest non-decreasing subsequence problem is to find the length of longest non-decreasing subsequences of $L$, denoted by LNDS(L). For example, if $L = [2, 2, 1, 7, 3, 8]$, then $[2, 2, 7, 8]$ is a longest non-decreasing subsequence of $L$ and LNDS(L) = 4. Please design an efficient algorithm to compute LNDS(L) and analyze its complexity.

3. (40 points) In a company, the supervisor-supervisee relation can be represented by a single tree $T$, with the president being the root of the tree. Given the tree $T$, you are asked to compute the maximal number of employees that can be invited to a party such that an employee and his/her immediate supervisor cannot be invited at the same time. Please design an efficient algorithm for this problem and analyze its time complexity.