Distributed Algorithms for Graph coloring

A few known results

- Any tree can be colored using two colors only
- Any graph whose maximum node degree is Δ can be colored using $(\Delta+1)$ colors
- Any planar graph can be colored using four colors, but no distributed algorithm is known and the centralized algorithm is also extremely cumbersome.
- Any tree of size n can be colored using three colors in $\log^*(n)$ rounds (Cole and Vishkin)

O(log*n) coloring of a tree

Cole & Vishkin's algorithm

Log*(n) is the *smallest number* of log-operations needed to bring n down to ≤2. For example, let n = one trillion. Now,

log (one trillion) = 40,

log (log (one trillion)) = 5.322, and

log (log (log (log (one trillion)))) < 2.

This means that log* (one trillion) = 4. This also illustrates that the function grows very slowly with the value of the argument.

O(log*n) coloring of a tree

Consider a rooted tree. The algorithm assumes that initially the color of each node is its id.

Each non-root node **v** is aware of its *parent* **p(v)** .

Interpret each color **c** as a *little-endian* bit string $c_{k-1}c_{k-2}\ c_{k-3} \ldots c_0$, and let **|c|** denote the size of the bit string.

O(log*n) coloring of a tree

program *reduce* for a rooted tree

The root node labels itself with color 0 followed by bit 0 of its old color;

{Program for each non-root node v in a round}

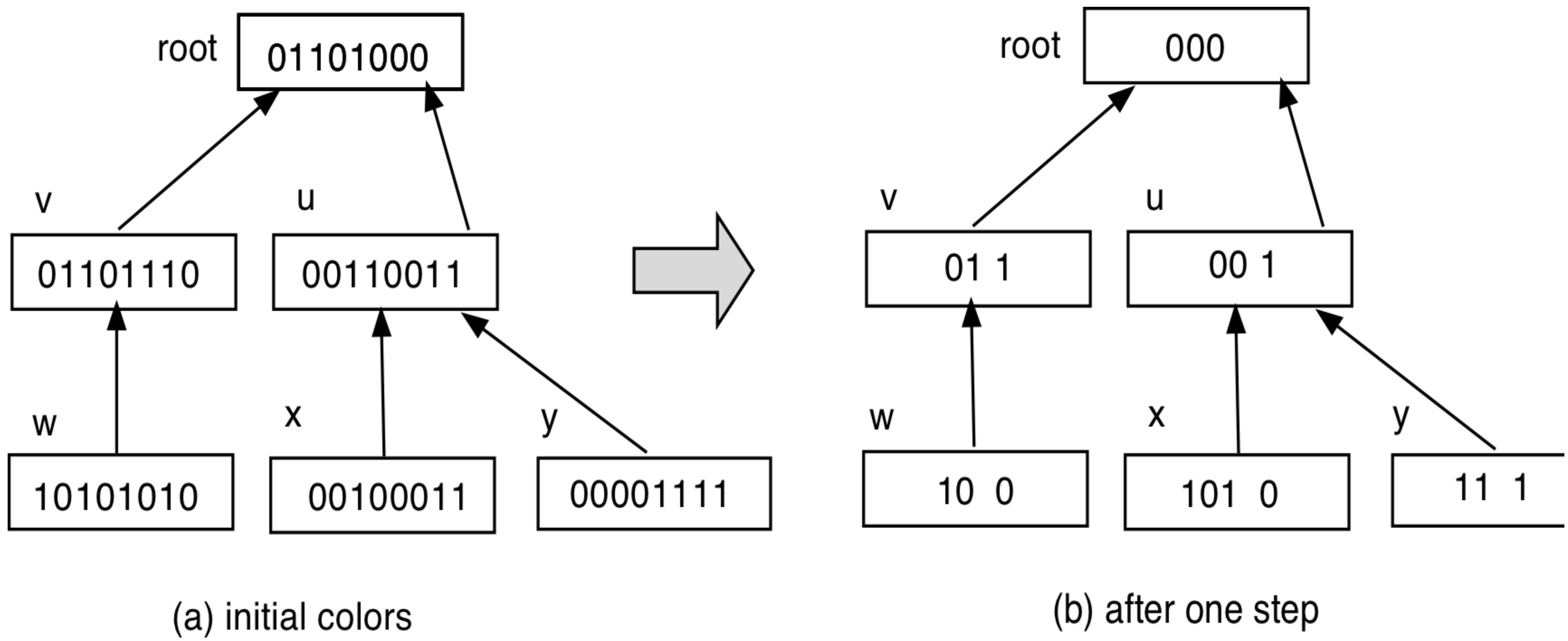
do $c(v) \geq 6 \rightarrow$

{Let j = smallest index where the bit strings of $c(v)$ and $c(p(v))$ differ}

new color $c(v) :=$ the bit string for j followed by bit j of $c(v)$

od

O(log*n) coloring of a tree



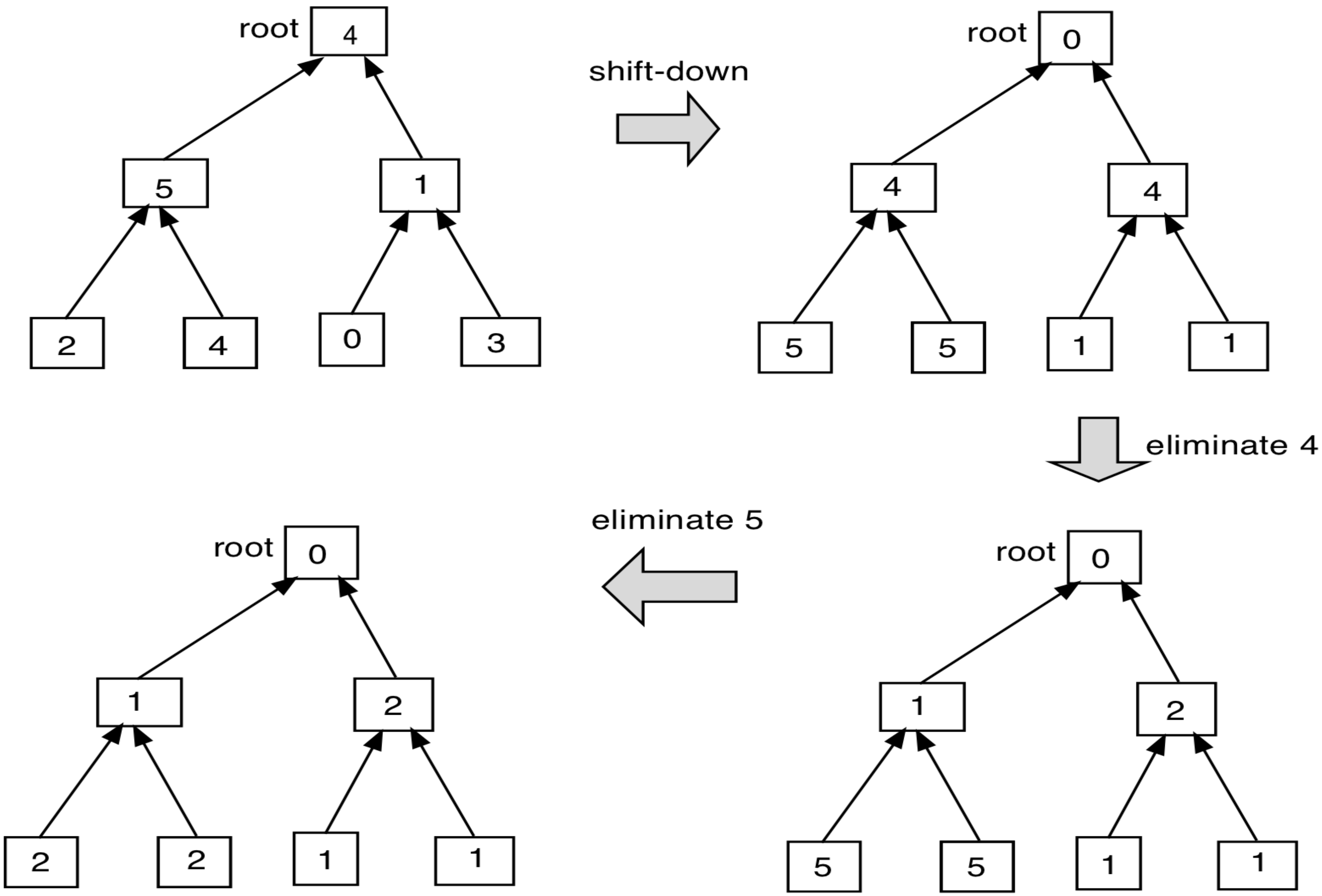
O(log*n) coloring of a tree

The shift-down operation

The root picks a new color < 6 and different from its current color, and

Each non-root node v concurrently executes $c(v) := c(p(v))$

O(log*n) coloring of a tree



O(log*n) coloring of a tree

Reduction of the color palette size from six to three

$z := 3$;

do $z \leq 5 \rightarrow$

$c(v) = z \rightarrow$ pick a color from {0, 1, 2} not used by the neighbors of v;

$z := z + 1$

od