Context-sensitive Examples

This document presents three examples of "context-sensitive" grammars. They are drawn from the book *Models of Computation and Formal Languages* by R. G. Taylor.

Example 11.1.2 The language L = $\{a^i b^j c^i d^j \mid i,j \geq 1\}$ is not context-free. There do exist grammars that generate L, however. Here is one:

$S \rightarrow AB$
$A \rightarrow aAX \mid aX$
$B \rightarrow bBd \mid bYd$
$Xb \rightarrow bX$
$XY \rightarrow Yc$
$Y \rightarrow \varepsilon$

The logic of the grammar is basically as follows. The productions for nonterminal A will be used to fix the value of i while the productions for nonterminal B will fix the value of j. Each occurrence of nonterminal X is replaced with terminal c but not before the production Xb → bX has been used to position it between the bs and ds. Nonterminal Y serves as a placemarker situated between the bs and the ds and can hence be used in positioning the Xs. The problem, of course, as the reader will likely have noticed, is that this grammar is not quite context-sensitive given the production for nonterminal Y (editor's note: Taylor defines a context-sensitive production $\alpha \rightarrow \beta$ to be one where len(α)≤len(β)). But it is fairly easy to obtain an equivalent grammar that is context-sensitive by adjusting the productions for S.

(i)-(ii)	$S \rightarrow aAB \mid aB$
(iii)-(iv)	$A \rightarrow aAX \mid aX$
(v)-(vi)	$B \rightarrow bBd \mid bYd$
(vii)	$Xb \rightarrow bX$
(viii)	$XY \rightarrow Yc$
(ix)	$Y \rightarrow c$

Placemarker Y is now itself being replaced with terminal c, so we do not need quite so many occurrences of nontermional X. A derivation of word aabbbccddd appears below.

$S \Rightarrow aAB$	by (i)
$\Rightarrow aaXB$	by (iv)
$\Rightarrow aaXXbBd$	by (v)
$\Rightarrow aaXbbBdd$	by (v)
$\Rightarrow aaXbbbYddd$	by (vi)
$\Rightarrow aabXbbYddd$	by (vii)
$\Rightarrow aabbXbYddd$	by (vii)

$\Rightarrow$ aabbbXYddd   by (vii)
$\Rightarrow$ aabbbYcddd   by (viii)
$\Rightarrow$ aabbbccddd   by (ix)

It should be apparent that every derivation of a word over $\Sigma = \{a,b,c,d\}$ will terminate with an application of production (ix). Moreover, by the time (ix) is used to eliminate nonterminal Y, all occurrences of nonterminal X must already have been converted to cs; once Y has disappeared. production (viii) will no longer be applicable.

Example 11.1.3 The language $L = \{a^i b^j c^k \mid 1 \leq i \leq j \leq k\}$ is not context-free. Again, There do exist grammars that generate L, however. One such grammar has the 10 productions.

$S \rightarrow aS'bX \mid abX$
$S' \rightarrow aS'bC \mid S'bC \mid S'C \mid bC \mid C$
$Cb \rightarrow bC$
$CX \rightarrow Xc$
$X \rightarrow c$

The productions for S and S**'** are first used to fix the values i, j, and k, in effect. Nonterminal X is introduced as a placemarker situated immediately to the right of all the bs. Precisely as in the preceding Example 11.1.2, occurrences of nonterminal C are moved to the right past the bs before being converted to cs. Finally, placemarker X is itself converted to c.

Example 11.1.4 We previously considered the theoretical linguist's use of context-free grammars in giving an account of the syntax of English. We close the present section with a brief discussion of the application within linguistics of context-sensitive rules. As an easy illustration, we describe the potential of such rules to interpret the phenomenon of subject-verb agreement with respect to number—that is, singular or plural—as reflected in sentences:

(a) The child runs.
(b) The men run.

To this end, we introduce the following rules (editor's note: syntax categories are S for sentence, NP for noun-phrase, VP for verb-phrase, N for noun, V for verb, and Det for determiner):

$S \rightarrow NP\ VP$
$NP \rightarrow Det\ N_{sing} \mid Det\ N_{plur}$
$N_{sing}\ VP \rightarrow N_{sing}\ V_{sing}$
$N_{plur}\ VP \rightarrow N_{plur}\ V_{plur}$
Det $\rightarrow$ **the**

$N_{sing} \rightarrow$ **child**
$N_{plur} \rightarrow$ **men**
$V_{sing} \rightarrow$ **runs**
$V_{plur} \rightarrow$ **run**

Note that the fourth and fifth rules here are context-sensitive but not context-free. Although it is possible to account for the subject-verb agreement using context-free rules only, the two context-sensitive rules capture neatly our intuition that the number of the subject determines that of the verb.